MATHEMATICS

Secondary I

NAME: ____

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1A

1. Basic Mathematics

"No number before zero. The numbers may go on forever, but like the cosmos, they have a beginning."

GIUSEPPE PEANO

1.1 Arithmetic Properties of Natural Numbers

In counting, we will use $1, 2, 3, 4, \cdots$. We usually call them as *natural numbers*. (We would denote them as $\mathbb{N}$.) If we include 0 in natural numbers, we get the *whole numbers*: $0, 1, 2, 3, \cdots$. There are two kinds of numbers in natural numbers.

Definition 1.1 — Even numbers. Even numbers are whole numbers that are divisible by 2.

Definition 1.2 — Odd numbers. Odd numbers are whole numbers that are not divisible by 2.

Then we discuss the arithmetic operations:

Operation	Symbol	Example	Description
Addition	$+$	$1+2=3$	1 *plus* 2 equals 3.
Subtraction	$-$	$4-3=1$	4 *minus* 3 equals 1.
Multiplication	$\times$	$5\times 6=30$	5 *times* 6 equals 30.
Division	$\div$	$8\div 2=4$	8 *divided by* 2 equals 4.

There is other description of expressions:

1. Add 10 to 2: $10+2$,
2. Subtract 2 from 5: $5-2$,
3. Multiply 3 by 2: 3×2,
4. Divide 4 by 2: $4\div 2$.

There are some rules in mixed operation calculations:

1. Perform multiplication and division if exists.
2. Perform the operation from left to right.

When multiplication and division exist together, then there is ambiguous in the calculation order. We then introduce *brackets*: When an expression contains brackets, we should perform the operations inside the brackets first.

Then we have an updated rule:

1. Perform operations inside brackets if exists.
2. Perform multiplication and division if exists.
3. Perform the operation from left to right.

Exercise 1.1 Find the result of "Divde 2 by the sum of 11 and 13, then add the quotient to 3".

1.2 Multiples and Factors

Definition 1.3 — Multiples. Multiples are natural numbers that we get when we multiply one natural number by another natural number.

Definition 1.4 — Factors. Factors are natural numbers such that the product of it and another natural numbers is a given number.

After introducing multiples and factors, we can introduce two special kinds of number.

Definition 1.5 — Prime Numbers. Prime number is a natural number except 1 where it has only 1 and itself as its factors.

Definition 1.6 — Composite Numbers. Composite number is a natural number except 1 where it has factors other than 1 and itself.

Prime numbers are useful in expressing a certain number with its factors in a concrete way.

■ **Example 1.1** The number 20 can be writen as $20 = 2 \times 2 \times 5$, where 2 and 5 are prime factors. ■

Before, we learnt the concept of *Highest Common Factor (H.C.F.)*, 最大公因數, and *Lowest Common Multiple (L.C.M)*, 最小公倍數.

Exercise 1.2 There are five cards of number 1, 2, 7, 8 and 9. Choose three cards to form a smallest three-digit number which is divisible by 8.

Exercise 1.3 Which of the following numbers is divisible by 9?

A. 80 145 B. 92 200 C. 102 131 D. 112 121

Exercise 1.4 Which of the following numbers is divisible by both 4 and 9?

A. 100 016 B. 214 380 C. 512 984 D. 975 622

Exercise 1.5 Which of the following numbers is divisible by 36?

A. 100 342 B. 431 210 C. 500 004 D. 987 214

Exercise 1.6 Find the H.C.F. and L.C.M. of 450 and 600 and leave the answers as prime factorisation. Hence find the H.C.F. and L.C.M. of ab^2c^2 and a^3bc^2.

Exercise 1.7 Find the prime factorisation of 198 and 693 by using short division. Hence express the H.C.F. and L.C.M. of 198 and 693 as prime factorisations.

Exercise 1.8 The H.C.F. and the L.C.M. of three numbers are 2×3^2 and $2^4\times3^5\times5^6$ respectively. If the first number and the second number are $2^2\times3^4\times5$ and $2^4\times3^2\times5^6$ respectively, then which of the followings is the third number?

A. 2×3^2 B. 2×3^5 C. $2\times3^2\times5$ D. $2\times3^5\times5$

1.3 Fractions and Decimals

Fractions are expressed as a numerator divided by denominator. For example, of $\frac{1}{4}$, 1 is the numerator and 4 is the denominator.

There are three types of fraction: Proper fractions (真分數), Improper fractions (假分數) and Mixed fractions (帶分數).

We can expand or reduce a fraction by multiply both the numerator and denominator by the same non-zero number or divide both of them by their common factor. Normally you are required to give your answer in the simplest form. But expanding a fraction is also important in the future.

A fraction can be expressed as decimals, such as $\frac{1}{4}=0.25$.

Then we can compare the values of any fractions by expanding the fractions or in decimal form.

There are also the same arithmetic properties of fractions as that of natural number. You learnt them in primary school.

Exercise 1.9 Find the value of $\left(3\frac{1}{2}-\frac{2}{3}\right)\div\left(\frac{4}{5}+\frac{1}{3}\right)$.

Exercise 1.10 Furina's monthly salary is \$33000. Every month, she spends $\frac{1}{10}$ of her salary on study, $\frac{1}{6}$ on food, $\frac{2}{5}$ on transport, and saves the rest. How much can Furina save in a year?

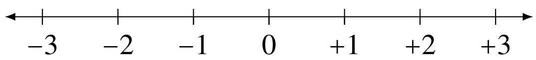
2. Directed Numbers and the Number Line

> "God made the integers, all else is the work of man."
>
> LEOPOLD KRONECKER

2.1 Concept of Directed Numbers

Directed numbers are the numbers with a positive '+' sign or a negative '−' sign. Note that 0 is neither a positive number nor a negative number, i.e., it is not a directed number. If any two directed numbers have the same magnitude but different sign, then they are called the opposite numbers. Then we have *integers*, where it includes natural numbers, their opposite numbers and the number 0. Natural numbers are also called as positive integers.

Directed numbers can be represented on a line in ascending order, and the line is called the number line. Numbers on the right is always bigger than numbers on the left.



Exercise 2.1 Write down the opposite number of $+2$.

Exercise 2.2 Arrange the following groups of numbers in ascending order with the help of a number line.

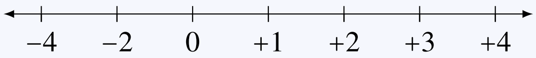
(1) $3, -4, +2$

(2) $-1, -\frac{1}{2}, -\frac{1}{4}, -\frac{1}{3}$

(3) $0.05, -2.13, -\frac{1}{4}, 1.01, -2, +2$

(4) $18, -3.14, +700, \frac{1}{64}, -\frac{1}{3}, -\frac{1}{2}, 0, 0.04$

Exercise 2.3 The number line in the following figure is not drawn correctly. What is the mistake of it?



A. There should not be arrows on the two ends of the line.
B. The negative numbers should be located on the right of zero.
C. The distance between any two consecutive integers should be equal.
D. The place of zero should be at the center.

2.2 Calculations of Directed Numbers

The calculations of directed numbers is just few simple steps. For addition and subtraction,

1. Consider a number x, a positive number $+a$ and a negative number $-a$,

$$x+(+a)=x+a \qquad x+(-a)=x-a$$
$$x-(+a)=x-a \qquad x-(-a)=x+a$$

2. Then adding a (positive) number will move to the right on the number line, and subtracting a (positive) number will move to the left on the number line.

Then for multiplication and division, consider positive numbers $+a, +b$ and negative numbers $-a, -b$,

$$(+a)\times(+b)=+(a\times b) \qquad \frac{(+a)}{(+b)}=+\left(\frac{a}{b}\right)$$
$$(+a)\times(-b)=-(a\times b) \qquad \frac{(+a)}{(-b)}=-\left(\frac{a}{b}\right)$$
$$(-a)\times(+b)=-(a\times b) \qquad \frac{(-a)}{(+b)}=-\left(\frac{a}{b}\right)$$
$$(-a)\times(-b)=+(a\times b) \qquad \frac{(-a)}{(-b)}=+\left(\frac{a}{b}\right)$$

Or in short, "正正得正，負負得正，正負得負，負正得負".

Exercise 2.4 Find the value of $(-2)[(-5)+(-6)(3)]$.

Exercise 2.5 Find the value of $(-15)-[(-8)-(+14)]$.

Exercise 2.6 Find the value of $\frac{1}{3}\div(-12)\div\frac{2}{9}$.

Exercise 2.7 If +2°C means an increase of 2°C in temperature, what are the meanings of

(a) −14°C, and (b) 0°C?

3. Introduction to Algebra

"If by chance I have omitted anything more or less proper or necessary, I beg forgiveness, since there is no one who is without fault and circumspect in all matters."

LEONARDO BONACCI (FIBONACCI)

3.1 Algebraic Language

Sometimes, we do not know a value but we know some relations about the value. Then, we use symbols or letters to represent the unknown values.

Similar to normal expression, expression with algebras are called *algebraic expressions*. All expressions are the same. Only algebraic multiplication have other types of expression: $a \cdot b$ and ab. We seldom use $\times$ in algebraic expression, as x is similar to $\times$, which may cause confusion.

For *index*, a^n is the same as "multiply a by itself n times". For example, $c \times c \times c = c^3$.

Exercise 3.1 Simplify the following expressions.

(a) $(x+x)(x+x+x+x)$
(b) $(2x-6x) \div 2+5x$
(c) $6\alpha \div 2-3\alpha \times (-4)$
(d) $15+9\star \div 3-\star -8$
(e) $y-2y \times 4+3y-4$
(f) $15a-2b-3a \times 3+5b$

3.2 Formulae and Method of Substitution

An equality that shows the relationship between two or more variables is called a *formula*. For example, $V = IR$ is a formula, where V, I and R are variables.

By the method of substitution, we can find the value of an unknown. To find a value in the formula with n variables, we needs to know at least $n-1$ variables. If less than $n-1$ variables is known, a new formula is derivated.

Exercise 3.2 Find the value of $S = a^2+b^2+c^2-2abc$ when $a=-2$, $b=-3$ and $c=-4$.

Exercise 3.3 There are 8 multiple choice questions in a quiz. 2 marks will be given for a correct answer. 1 mark will be deducted for a wrong answer. No marks will be deducted for leaving the question blank.

(a) Write down the formula for calculating the mark (M) scored by a student who answers x questions correctly, y questions wrongly and leaves z questions blank.

(b) Ayaka answered all the questions without leaving any questions blank, while Furina answered only 6 of the questions leaving 2 questions blank. If both of them answered 4 questions correctly, who got a higher score in the quiz? Explain your answer.

3.3 Sequence

A *sequence* is list of numbers. Each number in the sequence is called a *term*. We may denote each term by $T_1, T_2, \cdots, T_n$, where T_1 is the first term, T_2 is the second term, and so on. If a sequence follows a certain pattern, the nth term, T_n is called the *general term of the sequence*. It is an algebraic expression in terms of n.

There are some typical sequences:

1. Sequences of Polygonal Numbers
   (a) Sequence of Triangular Number: $1, 3, 6, 10, \cdots$. $T_n = 1+2+3+\cdots+n$.
   (b) Sequence of Square Number: $1, 4, 9, 16, \cdots$. $T_n = n^2$.
2. Arithmetic Sequences and Geometric Sequences
   (a) Arithmetic Sequence: $T_n = a+(n-1)d$, where a is the first term, d is the common difference.
   (b) Geometric Sequence: $T_n = ar^{n-1}$, where a is the first term, r is the common ratio.
3. Fibonacci Sequence: $T_1 = 1$, $T_2 = 1$, $T_n = T_{n-1}+T_{n-2}$.

Exercise 3.4 It is given that 22 and 18 are the 3rd term and the 4th term of an arithmetic sequence respectively. Find the 5th term and the 6th term of the sequence.

Exercise 3.5 It is given that the 11th term and the 12th term of the Fibonacci sequence are 89 and 144 respectively. Find the 13th term and the 14th term of the sequence.

Exercise 3.6 (a) Find the general terms of the following sequences: $1, 4, 9, 16, \cdots$; and $3, 6, 9, 12, \cdots$

(b) Consider the sequence $4, 10, 18, 28, \cdots$. Using the results of (a), find the general term of this sequence. Hence, find the 8th term of this sequence.

Exercise 3.7 Consider the sequence $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \cdots$.

(a) Find the general term of the sequence.

(b) Firefly claims that all the terms are smaller than 1. Do you agree? Explain your answer.

Exercise 3.8 It is given that $1+2+3+\cdots+n = \frac{n(n+1)}{2}$. Using the formula, find the values of

(a) $3+6+9+\cdots+120$,

(b) $101+102+103+\cdots+200$.

4. Algebraic Equations in One Unknown

"Here lies Diophantus, the wonder behold. Through art algebraic, the stone tells how old."

GREEK ANTHOLOGY

If an algebraic equation contains two expressions with one unknown connected by an equality sign, it is called an *algebraic equation in one unknown*. For example, $2x+1=8$. The value of the unknown that satisfies an equation is called the *solution* or the *root* of the equation. The values on both sides of an equation remain equal when the same operation is performed on both sides.

There are some techniques for solving equations: Transposition of terms; Combining terms with the same unknown; Removing brackets, and; Eliminating denominators.

Exercise 4.1 Solve the following equations.

(a) $\frac{3(u+5)}{4} = -6$

(b) $8x - \frac{1}{2} = 4 - \frac{5x}{2}$

(c) $12(4-x) = \frac{4}{3}(1-2x)$

(d) $-3(3x+2) = \frac{x}{3} - 6$

(e) $\frac{3x-5}{2} - x = -\frac{x}{3}$

(f) $\frac{2(x-3)}{9} - \frac{x}{6} = 2$

(g) $\frac{7-x}{11} - \frac{1-2x}{3} = x$

(h) $3[4(1-2d)+3d] = d$

(i) $\frac{2-r}{5} + \frac{r+1}{2} = \frac{2r}{5} - 1$

Exercise 4.2 (a) Solve the equation $4(x-9)+3(x-9)+2(x-9)+(x-9) = 55+5(x-9)$.

(b) Using the result in (a), solve the equation

$$4\left(1-\frac{2}{y}\right)+3\left(1-\frac{2}{y}\right)+2\left(1-\frac{2}{y}\right)+\left(1-\frac{2}{y}\right)=55+5\left(1-\frac{2}{y}\right)$$

Exercise 4.3 A ferry travels from pier P to pier Q. The travelling time downstream and upstream are 3 hours and 4 hours respectively. If the water flows at a speed of 1.5 km/h, find the distance between the two piers.

Exercise 4.4 A *Greek Anthology* about the ancient Greek mathematician Diophantus says:

"The boyhood of Diophantus lasted $\frac{1}{6}$ of his life; he married after $\frac{1}{7}$ more; his beard grew after $\frac{1}{12}$ more, and his son was born 5 years later; the son lived to half of his father's age, and finally Diophantus died 4 years after his son."

Based on this information, how old was Diophantus when he died?

Exercise 4.5 Yoimiya had an octopus card with a remaining value of \$250. She uses it only for everyday transportation. After 5 days the remaining value of her octopus card became \$130. If transportation fee is \$$x$ each day, write and equation in terns of x and solve it.

Exercise 4.6 Emilia pays a total of \$78 for 6 bottles of green tea and 4 bottles of coffee. The price of a bottle of coffee is \$2 more than that of a bottle of green tea. Find the price of a bottle of green tea.

Exercise 4.7 Kafka has 28 coins which include \$2 coins and \$5 coins only. If the total value of the coins is \$101, how many \$5 coins does he have?

Exercise 4.8 Ruby's pocket money is \$100 more than 3 times Aqua's pocket money. The total amount of their pocket money is \$500 more than twice Aqua's pocket money. Find the amount of Aqua's pocket money.

4.1 DSE Corner

4.1.1 Long Question

Problem 4.1 — HKDSE/2012/I/Q5.

There are 132 guards in an exhibition centre consisting of 6 zones. Each zone has the same number of guards. In each zone, there are 4 more female guards than male guards. Find the number of male guards in the exhibition centre.

5. Introduction to Geometry

"There is no Royal Road to Geometry."

EUCLID

5.1 Elements in Geometry

We would only talk about the Euclidean geometry (歐幾里得幾何), or more specifically in Cartesian coordinate system (笛卡兒坐標系).

Definition 5.1 — Points. A *point* is an ideal and exact position without size in a space and it is zero dimensional.

Definition 5.2 — Lines. A *straight line*, or *line*, is made up of infinite numbers of points which has length only but no width and it is one dimensional.

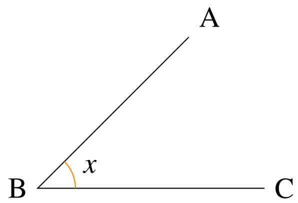
Definition 5.3 — Line Segments. A *line segment* is a part of straight line which has two end points and a fixed length.

Definition 5.4 — Planes. A *plane* is made up of infinite numbers of straight lines which has length and width and it is two dimensional.

5.2 Angles and Lines

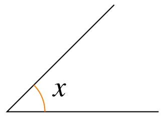
First we talk about how to name a simple angle.

Definition 5.5 — Angle. We called the angle ABC as $\angle ABC$ ($\angle CBA$) or x or $\angle B$ (not suggested).

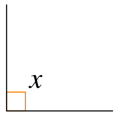


Then we have 6 difference types of angles.

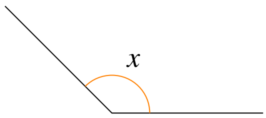
Definition 5.6 — Acute angle. The size is $0° < x < 90°$



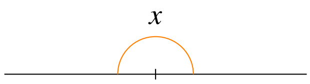
Definition 5.7 — Right angle. The size is $x = 90°$



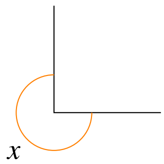
Definition 5.8 — Obtuse angle. The size is $90° < x < 180°$



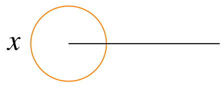
Definition 5.9 — Striaght Angle. The size is $x = 180°$



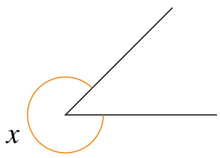
Definition 5.10 — Reflect Angle. The size is $180° < x < 360°$

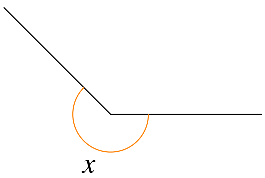


Definition 5.11 — Round Angle. The size is $x = 360°$



Note that the following two angles are reflect angles also.



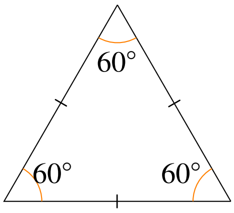
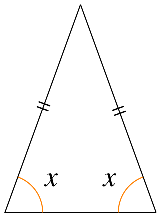
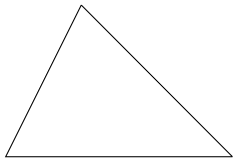


If two straight lines AB and CD on the same plane do not intersect no matter how far they extend, they are called *parallel lines*, denoted by $AB \mathbin{//} CD$.

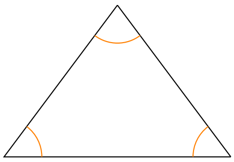
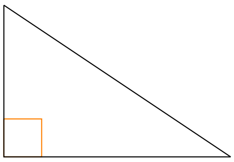
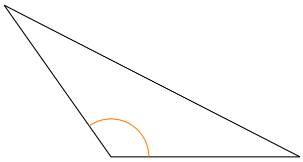
If two straight lines AB and CD intersect and form a right angle, they are called *perpendicular lines*, denoted by $AB \perp CD$.

5.3 Triangles, Circles and Polygons

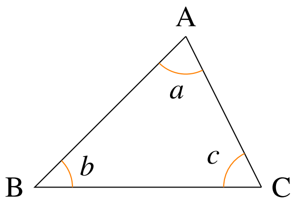
We have different types of triangles.

Equilateral triangle	Isosceles triangle	Scalene triangle
		

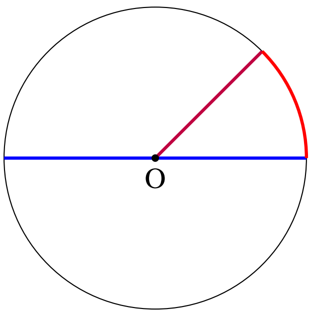
The above triangles are classified by the length of the edges. We can also classify them by angles.

Acute-angled triangle	*Right-angled triangle*	*Obtuse-angled triangle*
		

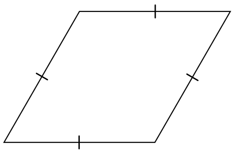
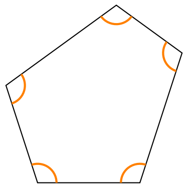
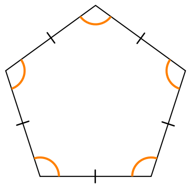
Definition 5.12 — Angle sum of triangle (Abbreviation: $\angle$ *sum of* $\triangle$). In $\triangle ABC$, $\angle A + \angle B + \angle C = a + b + c = 180°$.

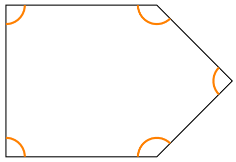
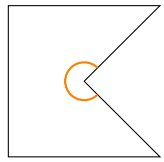


For circle, there are a few terms that you have to remember: diameter (直徑), radius (半徑) [Plural form: radii], centre (圓心), arc (弦), and circumference (圓周).



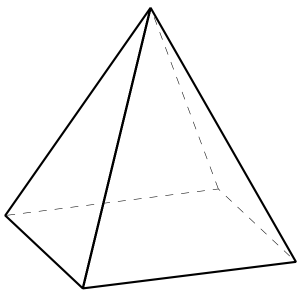
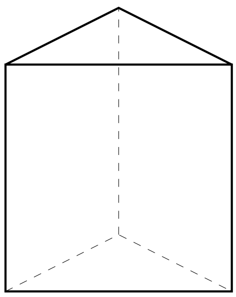
We have 5 types of polygons:

Equilateral	Equiangular	Regular
		

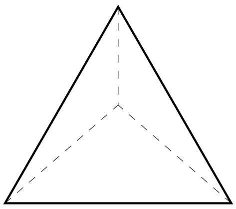
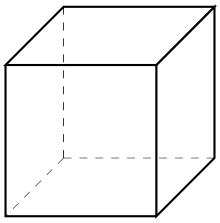
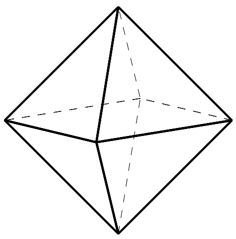
Convex	Concave
	

5.4 Three-Dimensional Figures

Polyhedra are solids enclosed by polygons.

Pyramid	Prism
	

A polyhedron having the same regular polygon on every face, with each vertex having the same number of edges connected to it, is called a *regular polyhedron* or a *Platonic solid*.

Regular Tetrahedron	Regular Hexahedron (Cube)	Regular Octahedron
		

For a polyhedron, we have an Euler's formula: $V+F-E=2$, where V, F and E are the number of vertices, faces and edges respectively.

If a solid is cut along a plane, the new end face obtained is a *cross-section of the solid*. If the cross-sections ofa solid have the same shape and size as its end faces when the solid is cut along anyplanes parallel to its end faces at different positions, then the solid is known as a *solid with uniform cross-section*.

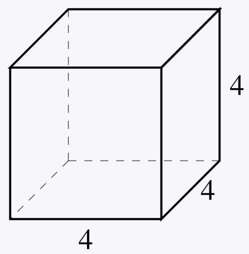
When we draw a solid on paper, we are using a plane figure to represent the solid. Such a plane figure is called the *2-D representation of the solid*. *Isometric grid paper* and *oblique grid paper* are two types of grid papers which are commonly used in drawing 2-D representation of solids.

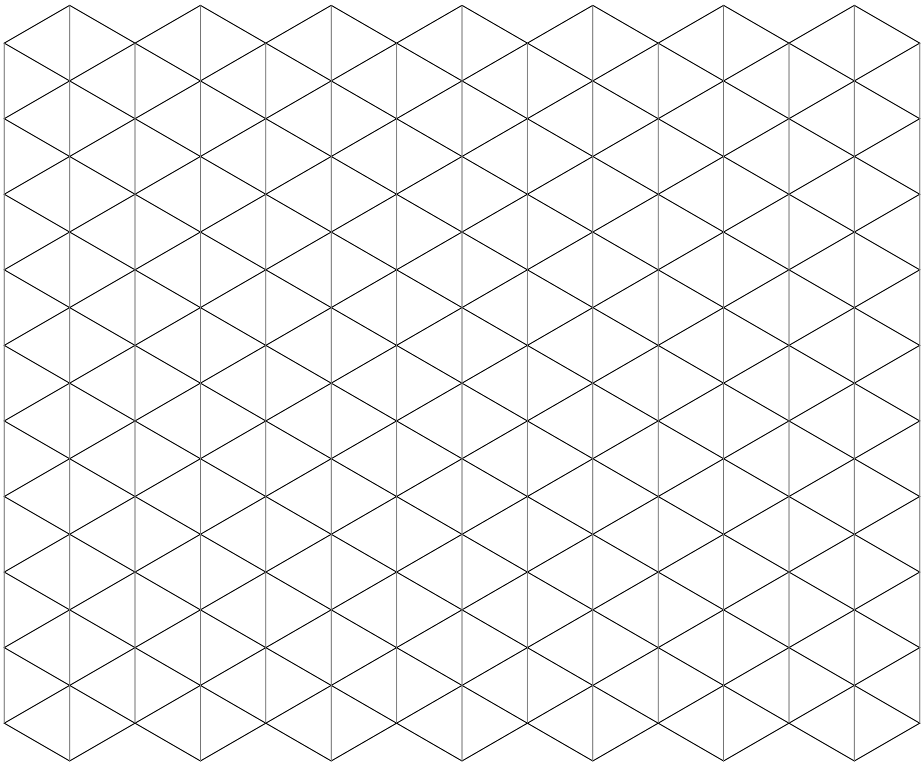
There are not much exercise, just memorise the terms.

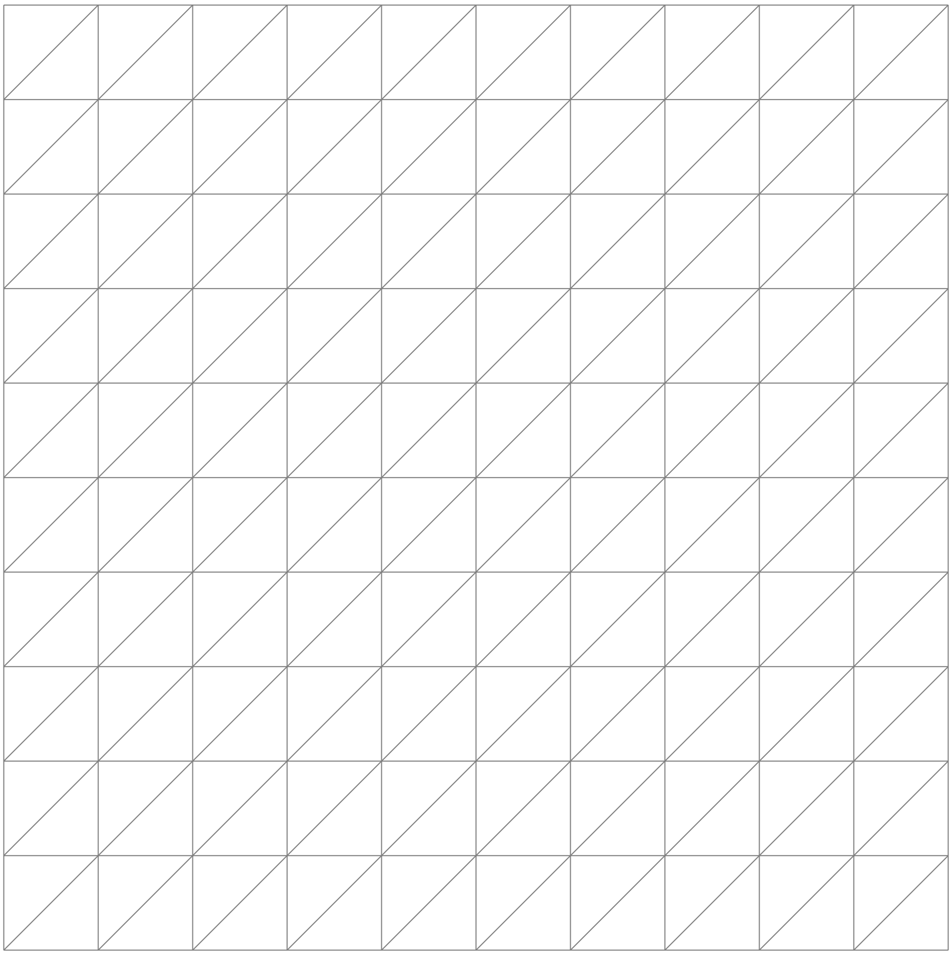
Exercise 5.1 Draw the 2-D representation of the solid below on each of the following grid paper.

(a) Isometric grid paper

(b) Oblique grid paper







6. Introduction to Statistics and Statistical Diagrams

"Lies, damned lies, and statistics."

BENJAMIN DISRAELI, 1ST EARL OF BEACONSFIELD

In statistics, we normally will perform analysis and draw conclusion only in exams but we should know about the process of getting a result.

Collection → Organisation → Presentation → Analysis → Conclusion.

There are some methods of collecting data

(a) Reading previous records
(b) Observation
(c) Interview
(d) Questionnaire
(e) Experiment

There are two types of data: *numerical* and *non-numerical*. For numerical data, there are *discrete data* and *continuous data*. See Exercise 6.1.

We can use frequency distribution table to organise data. See Exercise 6.2

Different statistical diagrams provide different information. The following table summarises the characteristics of different statistical diagrams.

Statistical Diagram	Characteristics
Bar chart (棒形圖)	show the actual frequency of each item
Pie chart (圓形圖)	show the percentage of each item
Broken-line graph (折線圖)	show the change in frequencies of data over a period of time and the trend of data
Stem-and-leaf diagram (幹葉圖)	present a small amount of data exactly
Scatter diagram (散點圖)	show the relationship between two quantities

For scatter diagram, there are three possible types of relationships between two sets of data: positive relationship, negative relationship or no obvious relationship.

Positive: When the value of x increases, the value of y increases, vice versa.

Negative: When the value of x increases, the value of y decreases, vice versa.

Exercise 6.1 Which of the following is a continuous datum?

A. Length
B. Number of days
C. Quantity of paper
D. Level of building

Exercise 6.2 Below are the numbers of children that 15 couples have.

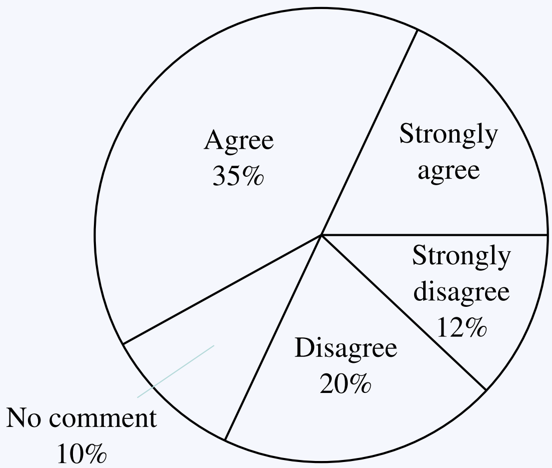
3, 0, 1, 0, 2, 2, 1, 0, 1, 0, 3, 0, 1, 0, 2

Construct a frequency distribution table for the above data.

Number of children	Tally	Frequency
0		
1		
2		
3		
Total		

Exercise 6.3 The pie chart below shows the opinions of 300 people on a new Government policy. How many people strongly disagree?

Options of 300 people on a new Government policy



Exercise 6.4 To show the changes in temperature of a city in one day, which of the following statistical graph should be used?

A. Broken line graph
B. Pie chart
C. Scatter diagram
D. Stem-and-leaf diagram

Exercise 6.5 The following stem-and-leaf diagram shows the heights of 20 plants.

Heights of 20 plants

Stem (10 cm)	Leaf (1 cm)							
0	a	8	9					
1	0	2	3	b	3	6	7	9
2	2	5	5	6	9	c		
3	0	2	5					

It is given that the difference between the heights of the tallest and the shortest plants is 28 cm. Then

A. $a = 3, b = 4, c = 9$

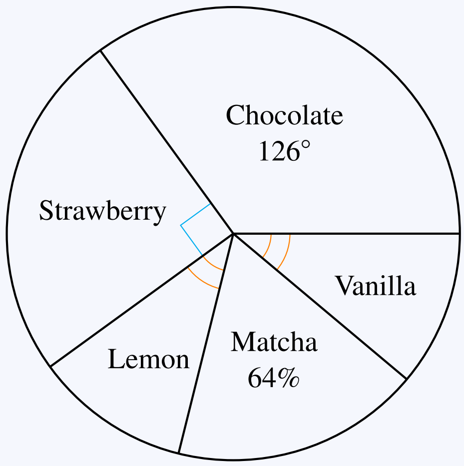
B. $a = 3, b = 4, c = 0$

C. $a = 7, b = 3, c = 9$

D. $a = 7, b = 3, c = 0$

Exercise 6.6 An ice cream shop earned \$900 in June. The following pie chart shows the distribution of the June income generated according to different ice cream flavours. The slice representing strawberry in the pie chart corresponds to a right angle.

Distribution of June income according to flavours



A. \$200

B. \$100

C. \$60

D. \$30

Exercise 6.7 Castorice recorded the seasonal revenue (in million \$) of her company last year.

Season	1	2	3	4
Revenue	18	22	15	8

Which of the following is the best to show the change in revenue last year?

A. Broken line graph

B. Pie chart

C. Scatter diagram

D. Stem-and-leaf diagram

Exercise 6.8 Which of the following data is suitable to be presented using stem-and-leaf diagrams?

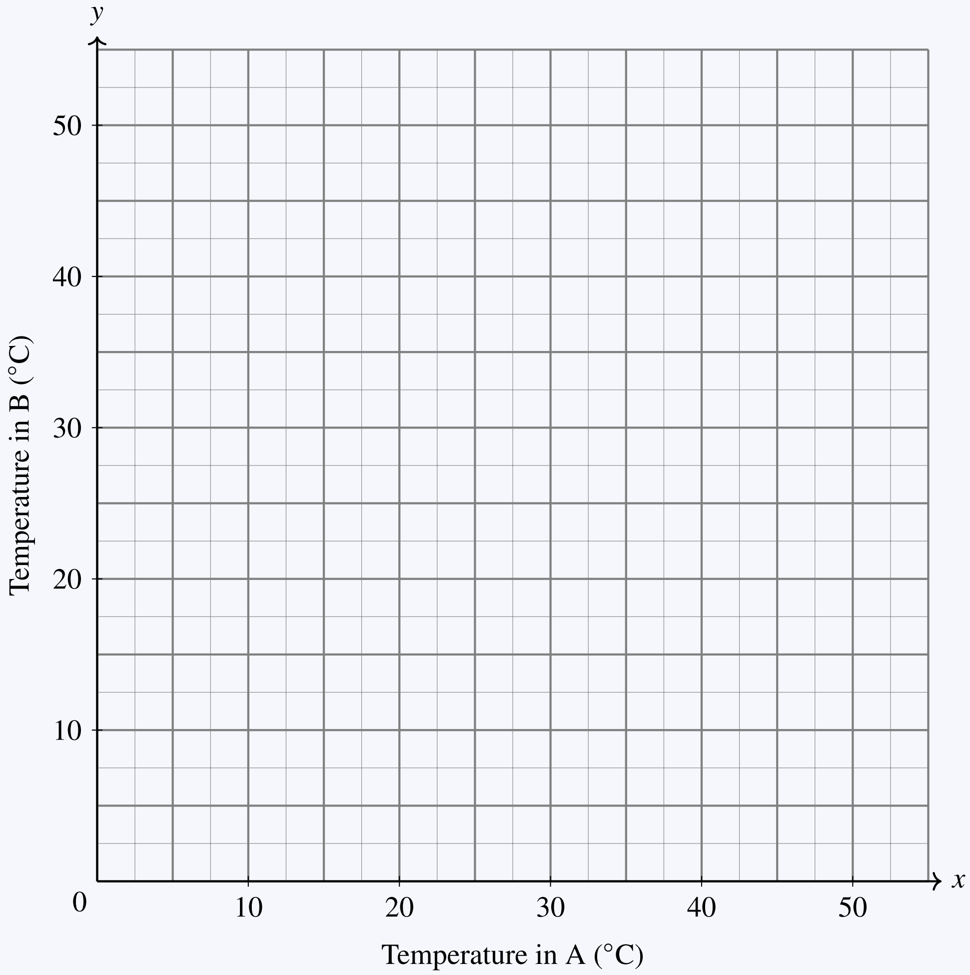
A. Blood types of 20 people.
B. Gender distribution in a city among different age groups.
C. Numbers of pages in 40 books.
D. Distances between each planet and the sun in the solar system.

Exercise 6.9 The following table shows the highest temperatures in City A and City B in the 12 months in 2024.

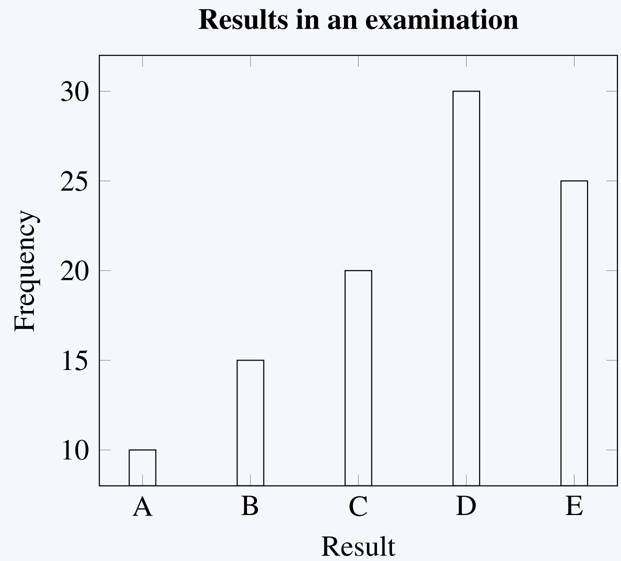
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
A (°C)	10	5	8	15	18	26	34	40	42	36	28	17
B (°C)	33	45	36	24	14	5	3	4	5	8	10	22

Using the data provided, construct a scatter diagram, and hence find the relationship between the highest temperatures of City A and City B.

The highest temperatures in 2 cities each of the 12 months in 2024



Exercise 6.10 The bar chart below shows the distribution of the results of 100 students in an examination. Find the percentage of students getting grade D or E.



A. 20%   B. 25%   C. 45%   D. 55%

II

1B

7. Percentage (I)

"Genius is 1% talent and 99% hard work."

ALBERT EINSTEIN

7.1 Percentage and Percentage Change

In the primary school, we have learnt some simple knowledge about *percentage*. Such as

$$\text{Percentage} = \frac{\text{part}}{\text{whole}} \times 100\%$$
$$\text{Part} = \text{whole} \times \text{percentage}$$

But not only we can use percentage to express the ratio between the part and the whole, we can also use it to express changes in values.

$$\text{Percentage change} = \frac{\text{new value} - \text{old value}}{\text{original value}} \times 100\%$$

If the *percentage change* is positive, then we have an increase in value. Similarly, for negative percentage change, we have a decrease in value. Note that *percentage decrease* normally will be using positive values to represent a decrease and negative to represent an increase, as we have mentioned the percentage is **decrease**.

One reason why percentage change is better than memorising *percentage increase* and percentage decrease is when we find the new value, using percentage change need not consider the sign, whether it is + or −.

$$\text{New value} = \text{old value} \times (1 + \%\ \text{change})$$

Exercise 7.1 Kiana gets 56 marks out of a total of 80 marks in a test. What percentage does she score?

Exercise 7.2 Stelle spent 75% of her income last month and saved $2500. Find her income last month.

Exercise 7.3 There were 2750 traffic accidents in a city in 2012. The number of traffic accidents increased by 6% in 2013, and decreased by 275 in 2014. What is the overall percentage change in the number of traffic accidents from 2012 to 2014?

7.2 Profit and Loss

For a shop, they will buy things from wholesalers and sell them to customers. The difference between the price bought from wholesalers, i.e. the *cost price*, and the price sold to the customers, i.e. the *selling price*, is the *profit*. For negative profit, we would use the term *loss* instead, and take its value only.

$$\text{Profit} = \text{selling price} - \text{cost price}$$

Similarly we can use percentage to express the degree of profit, and its name is *profit per cent*.

$$\text{Profit per cent} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

Also, we can use profit per cent and cost price to find the profit.

$$\text{Profit} = \text{cost price} \times \text{profit } \%$$

Corresponding to find the new value, we have

$$\text{Selling price} = \text{cost price} \times (1 + \text{profit } \%)$$

Exercise 7.4 Alya pays \$900 for 250 stickers. She sells them at \$3 each. What is the profit per cent?

Exercise 7.5 After 27 new members have joined a choir, the number of choir members is increased by 30%. Find the numbers of choir members respectively before and after the new members joined.

Exercise 7.6 A book is sold at \$57 in a shop and has made a loss of 5%. Find the cost price of the book and the loss when selling the book.

Exercise 7.7 The marked prices of a car in shop A and shop B are \$105 000 and \$110 000 respectively. Shop A offers a 3.5% discount and gets a 10% profit while shop B offers a 7% discount and gets a 5% profit. At which shop should Maria buy the car? Explain your answer.

Exercise 7.8 Last month, a company spent 50% of its total expense on rent 30% on staff salary, and the rest on other expenses. The total expense and profit last month was \$640 000 and \$26 880 respectively. This month, the company's rent decreases by 15%, staff salary increases by 18%. If the total expense remains unchanged, what is the percentage change in other expenses this month? If the amount of other expenses remains unchanged, what is the percentage change of the profit?

7.3 Discount

Things become complicated when *discount* is also considered. For any discount, there is a *marked price*, which stated the original price of the product to customers.

$$\text{Discount} = \text{marked price} - \text{selling price} = \text{marked price} \times \text{discount } \%$$

$$\text{Discount per cent} = \frac{\text{discount}}{\text{marked price}} \times 100\%$$

$$\text{Selling price} = \text{marked price} \times (1 - \text{discount } \%)$$

Exercise 7.9 On Valentine's Day, a flower shop offers 15% discount on all items. A bouquet of rose is sold for \$442. What is the marked price of the bouquet of rose?

Exercise 7.10 A supermarket buys chocolates at \$4 per pack and sells them at \$6 per pack. During a sales promotion, the super market has an offer of 'pay \$1 for the second pack of chocolates'. If Yuki buys 3 packs of chocolates, what is her overall discount per cent and the supermarket's over profit per cent on Yuki's purchase? If she bought 4 packs, how will the answer changed?

7.4 DSE Corner

7.4.1 Long Question

Problem 7.1 — HKDSE/SP/I/Q4.
The marked price of a handbag is \$560. It is given that the marked price of the handbag is 40% higher than the cost.

(a) Find the cost of the handbag.
(b) If the handbag is sold at \$460, find the percentage profit.

Problem 7.2 — HKDSE/PP/I/Q4.
The cost of a chair is \$360. If the chair is sold at a discount of 20% on its marked price, then the percentage profit is 30%. Find the marked price of the chair.

Problem 7.3 — HKDSE/2014/I/Q6.
The marked price of a toy is \$255. The toy is now sold at a discount of 40% on its marked price.

(a) Find the selling price of the toy.
(b) If the percentage profit is 2%, find the cost of the toy.

Problem 7.4 — HKDSE/2015/I/Q6.
The cost of a book is \$250. The book is now sold and the percentage profit is 20%.

(a) Find the selling price of the book.
(b) If the book is sold at a discount of 25% on its marked price, find the marked price of the book.

Problem 7.5 — HKDSE/2018/I/Q7.
The marked price of a vase is 30% above its cost. A loss of \$88 is made by selling the vase at a discount of 40% on its marked price. Find the marked price of the vase.

Problem 7.6 — HKDSE/2019/I/Q5.
A wallet is sold at a discount of 25% on its marked price. The selling price of the wallet is \$690.
(a) Find the marked price of the wallet.
(b) After selling the wallet, the percentage profit is 15%. Find the cost of the wallet.

Problem 7.7 — HKDSE/2021/I/Q6.
The marked price of a shirt is higher than its cost by \$80. The shirt is sold at a discount of 10% on its marked price. After selling the shirt, the percentage profit is 30%. Find the marked price of the shirt.

Problem 7.8 — HKDSE/2022/I/Q5.
A fan is sold at a discount of 30% on its marked price. After selling the fan, the profit is \$78 nad the percentage profit is 26%. Find the marked price of the fan.

7.4.2 Multiple Choice Question

Problem 7.9 — HKDSE/SP/II/Q10.
Mary sold two bags for \$240 each. She gained 20% on one and lost 20% on the other. After two transactions, Mary

A. lost \$20.
B. gained \$10.
C. gained \$60.
D. had no gain and no loss.

Problem 7.10 — HKDSE/PP/II/Q10.
John buys a vase for \$1600. He then sells the vase to Susan at a profit of 20%. At what price should Susan sell the vase in order to have a profit of 20%?

A. \$2 240
B. \$2 304
C. \$2 400
D. \$2 500

Problem 7.11 — HKDSE/2013/II/Q10.
Susan sells two carsfor \$80 080 each. She gains 30% on one and loses 30% on the other. After the two transactions, Susan

A. loses \$15 840.
B. gains \$5 544.
C. gains \$10 296.
D. has no gain and no loss.

Problem 7.12 — HKDSE/2014/II/Q9.
There are 792 workers in a factory. If the number of male workers is 20% less than that of female workers, then the number of male workers is

A. 352
B. 360
C. 432
D. 440

Problem 7.13 — HKDSE/2020/II/Q9.
The cost of a toy is x% lower than its selling price. After selling the toy, the percentage profit is 25%. Find x.

A. 20
B. 25
C. 75
D. 80

Problem 7.14 — HKDSE/2020/II/Q9.
The marked price of a jacket is 60% above its cost. A profit is \$104 is made by selling the jacket at a discount of 25% on its marked price. Find the cost of the jacket.

A. \$416 B. \$520 C. \$728 D. \$832

8. Approximate Values and Numerical Estimation

"All exact science is dominated by the idea of approximation."

BERTRAND ARTHUR WILLIAM RUSSELL,
3RD EARL RUSSELL, OM, FRS

8.1 Concept of estimation

In our world, we actually do not have an exact value of every stuffs, instead we would use approximations to express. For example, we cannot measure a string in the world with the length exactly 1 cm. The method of finding the approximate value of one quantity or a numerical expression is known as *estimation*. The approximate value obtained is called an *estimate*. Sometimes estimations are used mostly because of the following:

1. Exact value is not available,
2. To simplify calculations,
3. For easier understanding,
4. Exact value changes from time to time.

8.1.1 Numerical estimation strategies

There are 3 simple estimation strategies: *Rounding off* (四捨五入), *rounding up* and *rounding down*. Some may ask why there are rounding up and rounding down, you may refer to the following exercise to come up with the answer.

Sometimes instead of using one type of rounding off/up/down, we will use clustered value, which means the most representative number among the group of numbers. For example, the clustered value of 390, 398, 407, 411 is 400.

Similarly, when the number is so small, such as numbers below 5, when either rounding or clustered value is used, the "error" will be significant. (The concept of "error" will be learnt in Form 2) So we would use compatible numbers: a pair of approximated values that is easy to calculate. For instance, $1.33 \times 89.8 \approx 1\frac{1}{3} \times 90 = 120$. $1\frac{1}{3}$ and 90 are compatible numbers in this case.

For this strategy, translation strategy, I do not prefer to call it as one of the numerical "estimation" strategies but it is a good method in mental abacus. This strategy is by rearranging the order of operations to make the calculation easy, but the eastimations is also based on the above methods. For example, $432-148+23-251 \approx 430-150+20-250=(430+20)-(150+250)=50$. This is also suitable in multiplication and division.

Exercise 8.1 (a) Round up 749 121 to the nearest (i) hundred thousand, and (ii) ten thousand.
(b) Round down 1 079 659 to the nearest (i) thousand, and (ii) hundred.

Exercise 8.2 Round off the numbers to the nearest ten and estimate the values of the following.

(a) $84-32+77+96$, and (b) $873.93\times 32.5\div 31.978$,

Exercise 8.3 The number of students in Form 4, Form 5, and Form 6 are 127, 116, and 102 respectively. Estimate the total number of students by rounding off each number to the nearest ten.

Exercise 8.4 The length and the width of a rectangle are 18.09 cm and 25.95 cm respectively. Estimate the area of the rectangle by rounding off the length and the width to the nearest cm.

Exercise 8.5 Kafka, Firefly, Silver Wolf and Blade have \$69.7, \$85.4, \$97.2 and \$70.8 respectively.
(a) Estimate the total amount they have by
 (i) rounding up each amount to the nearest dollar,
 (ii) rounding down each amount to the nearest dollar.
(b) The marked price of a birthday gift is \$400 and it is sold at 15% off. Do they have enough money to buy the birthday gift if they put their money together? Explain your answer.

Exercise 8.6 Correct 243.56 to the nearest unit.

A. 240 B. 243 C. 243.6 D. 244

Exercise 8.7 Correct 50.997 to 2 decimal places.

A. 50.90 B. 50.99 C. 51.00 D. 51

Exercise 8.8 Which of the following is/are an estimate(s)?
I. A city's number of births is 57 000 in 2025.
II. The speed of a plane is 1000 km/h.
III. A calculator has a display of 10 digits.

A. I only
B. III only
C. I and II only
D. II and III only

Exercise 8.9 The weights of four puppies are measured to be 2.5 kg, 4.1 kg, 3.7 kg and 6.3 kg. Find the average weight of these 4 puppies by rounding off the measurements correct to the nearest kg.

A. 4 kg B. 4.15 kg C. 4.25 kg D. 4.5 kg

Exercise 8.10 Which of the following pairs of compatible numbers can be used to estimate the value of 0.665×81.2?

A. 0.7 and 82 B. 0.6 and 81 C. $\frac{3}{5}$ and 82 D. $\frac{2}{3}$ and 81

Exercise 8.11 Round down 769.54 to the nearest integer.

A. 769.00 B. 769.0 C. 769 D. 770

Exercise 8.12 0.179 28 =

A. 0.1 (correct to 1 decimal place)
B. 0.17 (correct to 2 decimal places)
C. 0.18 (correct to 3 decimal places)
D. 0.179 (correct to 3 decimal places)

Exercise 8.13 Estimate the value of $341.1 + 59.7 \times 84.6$ by rounding up each number to the nearest ten.

A. 5 390 B. 5 400 C. 5 740 D. 5 780

Exercise 8.14 The lengths of 4 ropes are 164 cm, 198 cm, 241 cm and 175 cm respectively. Estimate the average length of the 4 ropes by rounding down each length correct to the nearest 10 cm.

A. 190 cm B. 194.5 cm C. 195 cm D. 200 cm

Exercise 8.15 The thickness of a dictionary is 10.5 cm and equals 2 piles of paper. Each pile has 500 pieces of paper. Estimate the thickness of a piece of paper.

A. 0.21 mm B. 0.105 mm C. 0.021 mm D. 0.0105 mm

8.2 Concept of measurement

As I have mentioned, in real world, there is no exat measurements. All we have is approximations only. Appropriate measuring tool, unit and degree of accuracy for measurements should be precisely considered. To reduce errors in measurement, we can

1. use measuring tools with finer scale intervals,
2. take readings correctly, and
3. take repeated measurements and use the average value.

8.2.1 Estimation strategies in measurement

When we do not have any measurement tools in hand, we could use the following strategies to help.

1. Benchmark strategy
2. Decomposition-recomposition strategy
3. Measuring groups of objects
4. Using formulae

Exercise 8.16 What if the following is the most appropriate unit for the volume of an apple?

A. mm^3 B. cm^3 C. m^3 D. L

Exercise 8.17 Herta wanted to estimate the length of the school hall. She first measured her hand span with a ruler, then used it as a benchmark to estimate the length of the school hall. However, she found that the difference between her estimation and the actual length was large.

I. Use a longer ruler to measure the hand span
II. Take the reading from the ruler at a correct line of sight.
III. Use the length of a benchmark instead of hand span.

A. I only
B. II only
C. II and III only
D. I, II and III

8.3 Significant figures

Instead of the exact values, we normally take the approximated values in application, such as Physics, and Chemistry. In taking approximation, we use a term called *significant figures*, short form sig. fig., which shows the important digits in a number.

The first significant figure in a number is the non-zero digit that has the greatest place value. Digits subsequent to the first significant figure are called the second, the third, $\cdots$ significant figures.

For example, the first significant figure of 0.01032 is 1 in the hundredth place, while the second significant figure is 0 in the thousandth place. Note that only the first significant figure cannot be zero but others can be. The zeros before the first significant figure are called *place holders*.

Exercise 8.18 Aglaea earns \$120 949 a month. Find her money earned in a year correct to 3 sig. fig.

Exercise 8.19 In a wet market, one fish is sold at \$14.5. Nilou pays for 6 fishes with a \$100 note.

(a) Estimate the change by rounding up the price of each fish to 2 significant figures.
(b) Is the actual change higher than, lower than or equal to the answer in (a)?

Exercise 8.20 What is the third significant figure in 5.012 346.

A. 6 B. 5 C. 2 D. 1

Exercise 8.21 Round off 674.996 to 5 significant figures.

A. 700.00 B. 674.99 C. 675.00 D. 675

Exercise 8.22 $\frac{58}{113} =$

A. 0.49 (correct to 2 significant figures)
B. 0.5 (correct to 2 significant figures)
C. 0.50 (correct to 2 significant figures)
D. 0.496 (correct to 2 significant figures)

Exercise 8.23 If $0.8544 < x < 0.8549$, which of the following is true?

A. $x = 0.8$ (correct to 1 decimal place)
B. $x = 0.8$ (correct to 1 significant figure)
C. $x = 0.85$ (correct to 2 decimal places)
D. $x = 0.854$ (correct to 3 significant figures)

Exercise 8.24 Estimate the value of $2678 - 1667 - 394 + 3872$ by rounding down each number to 2 significant figures.

A. 4 400 B. 4 500 C. 4 410 D. 4 490

8.4 DSE Corner

8.4.1 Long Questions

Problem 8.1 — HKDSE/2014/I/Q3. (a) Round up 123.45 to 1 significant figure.
(b) Round off 123.45 to the nearest integer.
(c) Round down 123.45 to 1 decimal place.

Problem 8.2 — HKDSE/2018/I/Q3. (a) Round up 265.473 to the nearest integer.
(b) Round down 265.473 to 1 decimal place.
(c) Round off 265.473 to 2 significant figures.

Problem 8.3 — HKDSE/2020/I/Q3. (a) Round up 534.7698 to the nearest hundred.
(b) Round down 534.7698 to 2 decimal places.
(c) Round off 534.7698 to 2 significant figures.

8.4.2 Multiple Choice Questions

Problem 8.4 — HKDSE/PP/II/Q14.
$0.009049999 =$

A. 0.00905 (correct to 3 decimal places)
B. 0.00905 (correct to 3 significant figures)
C. 0.00905 (correct to 6 decimal places)
D. 0.00905 (correct to 6 significant figures)

Problem 8.5 — HKDSE/2012/II/Q13.
$0.0322515 =$

A. 0.032 (correct to 3 significant figures)
B. 0.0322 (correct to 4 decimal places)
C. 0.03225 (correct to 5 significant figures)
D. 0.032252 (correct to 6 decimal places)

Problem 8.6 — HKDSE/2013/II/Q4.
$0.0504545 =$

A. 0.051 (correct to 2 significant figures)
B. 0.0505 (correct to 3 decimal places)
C. 0.05045 (correct to 4 significant figures)
D. 0.05046 (correct to 5 decimal places)

Problem 8.7 — HKDSE/2015/II/Q4.
$0.0023456789 =$

A. 0.00235 (correct to 6 decimal places)

B. 0.002345 (correct to 6 decimal places)
C. 0.002346 (correct to 6 significant figures)
D. 0.00234568 (correct to 6 significant figures)

Problem 8.8 — HKDSE/2016/II/Q4.
0.0765403 =

A. 0.076 (correct to 2 significant figures)
B. 0.0765 (correct to 3 decimal places)
C. 0.07654 (correct to 4 significant figures)
D. 0.076540 (correct to 5 decimal places)

Problem 8.9 — HKDSE/2017/II/Q4.
$\frac{1}{\pi^4} =$

A. 0.0102 (correct to 3 significant figures)
B. 0.01025 (correct to 4 significant figures)
C. 0.01026 (correct to 5 decimal places)
D. 0.010266 (correct to 6 decimal places)

Problem 8.10 — HKDSE/2019/II/Q6.
If $0.06557 < x < 0.06564$, which of the following is true?
A. $x = 0.065$(correct to 2 decimal places)
B. $x = 0.065$ (correct to 2 significant figures)
C. $x = 0.0656$ (correct to 3 decimal places)
D. $x = 0.0656$ (correct to 3 significant figures)

9. Areas and Volumes (I)

"The only teaching that a professor can give, in my opinion, is that of thinking in front of his students."

HENRI LÉON LEBESGUE FORMEMRS

9.1 Areas

In primary school, we have learnt areas of some simple plane figures. The area of

1. Squares = side × side = side^2
2. Rectangles = length × width
3. Triangles = $\frac{1}{2} \times \text{base} \times \text{height}$
4. Parallelograms = base × height
5. Trapeziums = $\frac{1}{2} \times (\text{upper base} + \text{lower base}) \times \text{height}$

Other than simple polygons, like squares and rectangles, there are more polygons where many of them can be split into several simple polygons like squares and rectangles which you know how to calculate them. There are two methods to find an area of a polygon:

1. Splitting method
2. Filling method

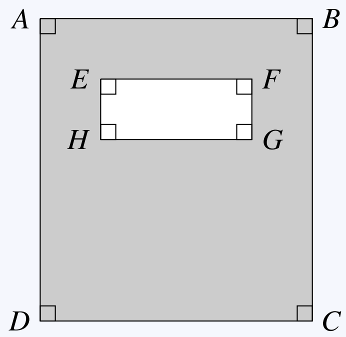
Exercise 9.1 Elysia has a kite of area 18 cm^2. Mei also has one and its area is larger than Elysia's by 600 mm^2. Find the area of Mei's kite.

A. 24 cm^2 B. 78 cm^2 C. 618 mm^2 D. 780 mm^2

Exercise 9.2 The length and the perimeter of a rectangle are 4 cm and 16 cm respectively. Find the area of the rectangle.

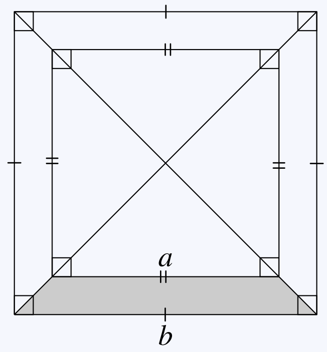
A. 8 cm^2 B. 16 cm^2 C. 32 cm^2 D. 64 cm^2

Exercise 9.3 In the figure, $AB = 9$ cm, $BC = 10$ cm, $EF = 5$ cm and $FG = 2$ cm. Find the area of the shaded region.



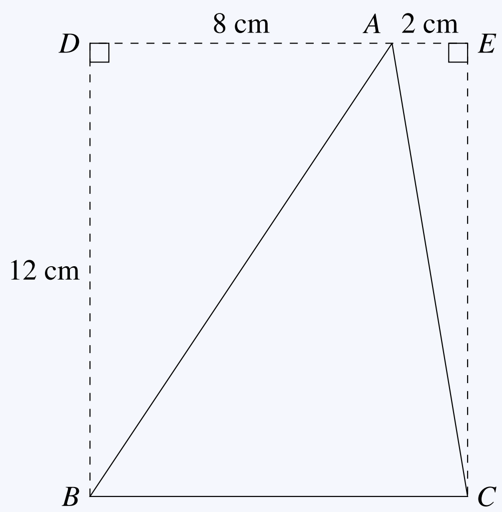
A. 70 cm^2 B. 75 cm^2 C. 80 cm^2 D. 85 cm^2

Exercise 9.4 In the figure, $a = 6$ cm and $b = 9$ cm. Find the area of the shaded region.



A. 9.25 cm^2 B. 9.75 cm^2 C. 10.25 cm^2 D. 11.25 cm^2

Exercise 9.5 Find the area of $\triangle ABC$.



Exercise 9.6 Which of the following is/are true?

I. If the perimeters of two rectangles are the same, their areas are also the same.

II. If the width of rectangle A is shorter than that of rectangle B, the area of rectangle A is larger than that of rectangle B.

III. If the product of the length and width of rectangles A and B are the same, their areas are also the same.

A. II only

B. III only

C. I and III only

D. II and III only

Exercise 9.7 The length of rectangle $ABCD$ is longer than its width by 8 cm. If the length of rectangle $ABCD$ is 14 cm, find the area of rectangle $ABCD$.

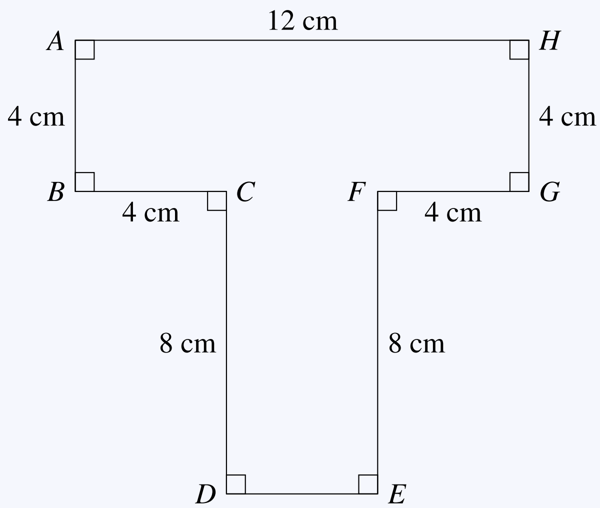
A. 42 cm^2

B. 84 cm^2

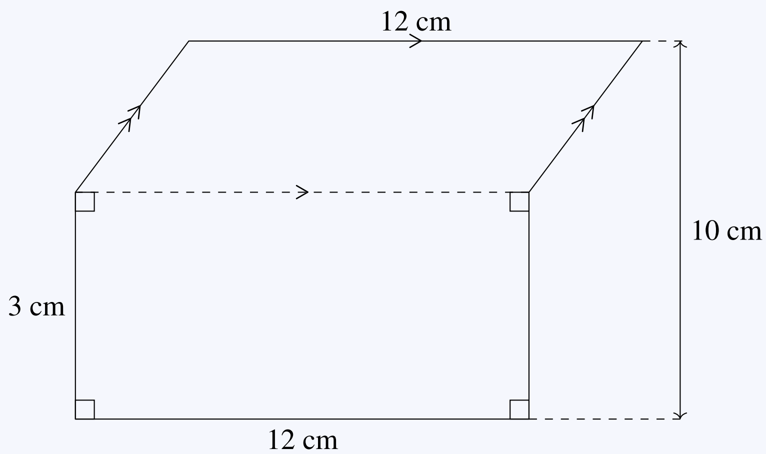
C. 176 cm^2

D. 308 cm^2

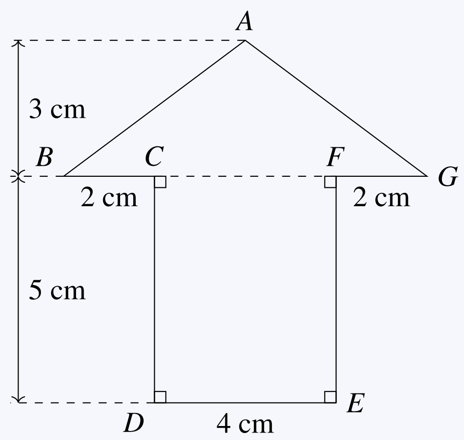
Exercise 9.8 Find the area of polygon ABCDEFGH.



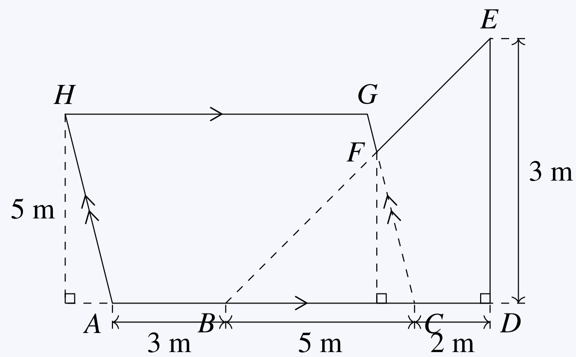
Exercise 9.9 Find the area of the polygon.



Exercise 9.10 Find the area of polygon ABCDEFG.



Exercise 9.11 Find the area of polygon ABCDEFGH.



9.2 Volumes

In primary school we have also learnt volumes of cuboids and pyramids. The volume of

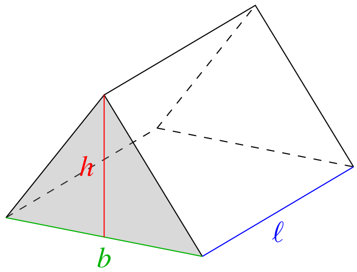
1. Cuboids = base area × height
2. Pyramids = $\frac{1}{3}$ × base area × height

One specific object is cube, where all sides are equal, then the volume can be written as side^3.

Similar to cuboids, there is one called prism.

$$\text{Volume of a prism} = \text{cross section area} \times \text{height} = \text{base area} \times \text{height}.$$

Note that cuboids are also called as rectangular prisms. One thing to be noted is, unlike cuboids you can take any face as your base, you must find its cross sections. Such as the following triangular prism.



Other than the cross section area, a new term called the surface area is introduced. As its name suggests, the surface area is the area of the surface of the 3D object. We know how to calculate the base area of a prism, but the "side" is a bit complicated as the edges of the base area increase, the number of "side" areas increases. Is there an easy way to calculate the "side" area? Indeed it is, just imagine combining the "side" areas together and we get the width of the plane is the height of prism and the length is the perimeter of the base. We called the "side" area as lateral surface area.

$$\text{Lateral surface area of a prism} = \text{perimeter of the base} \times \text{height},$$
$$\text{Total surface area of a prism} = \text{base area} \times 2 + \text{lateral surface area}.$$

10. Manipulation of Simple Polynomials

"Mathematics is the queen of the sciences,
and arithmetic the queen of mathematics."

JOHANN CARL FRIEDRICH GAUSS

10.1 Law of positive integral indices

In Chapter 3, we have already learnt the concept of index, such as $c \times c \times c \times c = c^4$. Then there is a law for multiplying or dividing two same base number but with different indices. For simiplifications, I would introduce a more well-defined version of the law.

Definition 10.1 If both m and n are integers, then

(a) $a^m \times a^n = a^{m+n}$

(b) $a^m \div a^n = a^{m-n}$, where $a \neq 0$

(c) $a^m = \dfrac{1}{a^{-m}}$, where $a \neq 0$

Exercise 10.1 $9b^{22} \div (-81b^{11}) =$

A. $-\dfrac{b^2}{72}$. B. $-\dfrac{b^2}{9}$. C. $-\dfrac{b^{11}}{72}$. D. $-\dfrac{b^{11}}{9}$.

Exercise 10.2 Simplify $14a^5 \times 11a \div 196a^4$.

A. $\dfrac{14a^2}{11}$ B. $\dfrac{14}{11a^2}$ C. $\dfrac{11a^2}{14}$ D. $\dfrac{11}{14a^2}$

Exercise 10.3 Simplify the following expressions.

(a) $x^3 \times x^2$

(b) $5x \times 3x^2$

(c) $2a^2 \times 4a^3$

(d) $10y^4 \div 2y^2$

(e) $8m^5 \times 2m^3 \div 4m^2$

(f) $12x^2 \div 3x \times 2x^3$

(g) $15p^3 \times 4p^2 \div 5p$

(h) $6z^4 \times z^2 \div 3z^3$

(i) $9k^5 \div 3k^2 \times 6k^3$

(j) $25x^4 \div 5x^2 \times 2x^3$

10.2 Polynomials

10.2.1 Monomials

Before getting into the polynomial world, we first consider the monomial case. Mono- means one, so monomial means there is only one term in an algebraic expression. It can be a number, a variable or the product of multiple numbers and variables. Not every "one term" with variables is a monomial, there is some exercises testing the concept of monomial.

There is two important concepts about monomials and, of course, polynomials.

Definition 10.2 — Coefficient. Coefficient of a monomial is the numerical part.

Definition 10.3 — Degree. Degree of a monomial is the sum of exponents of all variables.

Example 10.1 For the monomial $81x^2y^2$, its coefficient = 81 and its degree $= 2+2 = 4$. ■

10.2.2 Polynomials

Poly- means multiple. So we know that polynomial is actually made up from multiple monomials by additions and subtractions. Note that monomial is also a polynomial. In monomials, we would call it as one term only, where number of terms means number of monomials, such as x^2+y^2 is 2 terms. The degree of a polynomial is the highest degree of all terms in a polynomial.

Example 10.2 For the polynomial $8xy+x+y+2$, the degree of the polynomial = 2 and the constant term = 2. ■

A polynomial with only one variable is called a *polynomial in one variable*. The terms are usually arranged in descending powers of the variable, such as x^3+3x^2+2x+1. Of course, you can arrange in ascending order. If there is two variables, we will normally arrange them in descending powers of each variable and the degree of each term, such as $x^2+y^2+2xy+x+y+1$.

If any two terms that have the *same variable(s)* to the *same power(s)*, they are called *like terms*. Otherwise, they are *unlike terms*. Like terms can be simplified by adding or subtracting their coefficients.

When you have the values of all variables in a polynomial, you can calculate the value of the polynomial by substitution. Sometimes you do not need the explicit values, such as $x = 2$ and $y = 4$, but the implicit values, such as $x+y = 6$ and $x-y = -2$, may also solve the polynomials.

Exercise 10.4 Find the value of the polynomial $\left(x^2+y^2-1\right)^3-x^2y^3$ when $x = -1$ and $y = 1$.

A. -1 B. 0 C. 1 D. 2

Exercise 10.5 Find the value of the polynomial $3x^2-4y+5$ when $x = 2$ and $y = 1$.

A. 9 B. 11 C. 13 D. 15

Exercise 10.6 Which of the following is NOT a polynomial?

A. $\frac{2}{3}\alpha+23-21$ B. $\frac{\alpha}{\beta}+9$ C. $-\frac{8}{9}$ D. $\frac{\alpha^2}{5}-\frac{\beta^2}{7}$

Exercise 10.7 Which of the following is NOT a polynomial?

A. $3x^2+2x-\frac{5}{3}$ B. $\frac{1}{3}\sqrt{x}+4$ C. $\frac{7y^3}{3}-2y+1$ D. $-\frac{1}{2}x^4+3x^2$

Exercise 10.8 Which of the following is/are correct?

I. $3x^9-7y^2x+3$ is a polynomial, but $3x^9$ is not a polynomial.
II. $-2y^3xz^4$ and z^4xy^3 are like terms.
III. -999 is not a monomial.

A. II only
B. III only
C. I and III only
D. I, II and III

Exercise 10.9 For the polynomial $4x^2+9+(-2)x^3+8x$,

(a) write down the constant term and the degree of the polynomial; and
(b) arrange the terms in ascending powers of x.

10.2.3 Manipulations in Polynomials

Similar to numbers, you can add or subtract any polynomials and group the like terms together. For the muliplications, we need to use the *distributive law of multiplication*: $a(x+y)=ax+ay$ or $(x+y)a=xa+ya$.

■ **Example 10.3** $(8x+y)-(2x+2y)=8x+y-2x-2y=(8-2)x+(1-2)y=6x-y$. ■

■ **Example 10.4** $(8x+y)(2x+2y)=8x(2x+2y)+y(2x+2y)=16x^2+16xy+2xy+2y^2=16x^2+18xy+2y^2$. ■

R Note that $-(x+y)=(-1)(x+y)=(-1)x+(-1)y=-x-y$ and $-(x-y)=(-1)(x-y)=(-1)x-(-1)y=-x+y$.

Exercise 10.10 Simplify $12y+(4y-2y)$

A. $10y$ B. $12y$ C. $14y$ D. $16y$

Exercise 10.11 Simplify $(5a+12)-(3a-4)$

A. $2a+16$ B. $8a+8$ C. $2a-8$ D. $8a+16$

Exercise 10.12 Simplify $(3y^2-4y+5)+(2y^2+7y-3)$

A. $5y^2+2y+8$ B. $5y^2+3y+1$ C. $5y^2+3y+2$ D. $5y^2+4y+2$

Exercise 10.13 Simplify $(2z^3-3z^2+4z)+(5z^3+z^2-6)-(z^3-2z+3+4z^2)$

A. $6z^3-6z^2+6z-9$
B. $6z^3-6z^2+z-3$
C. $8z^3-6z^2+6z-9$
D. $8z^3-6z^2+z-3$

Exercise 10.14 Simplify the following algebraic expressions:

(a) $2x+3y-(y-4)$
(b) $4a-(2a+5)$
(c) $5h^2+2h-(3h^2-4h+1)$
(d) $10m^2+4m-(5m^2-2m+1)$
(e) $9x^2+5x-(4x^2+3x-2)$
(f) $12y^3-(3y^3+4y^2-6)$

Exercise 10.15 Which of the following algebraic expressions is equivalent to $-3y^5$?

A. $-3+y^7-y^2$ B. $-3-y^2-y^3$ C. $-3y^7y^{-2}$ D. $-3(-y^5)$

Exercise 10.16 Expand $(2x+y)(3y-x)$.

A. $3y^2+2x^2-7xy$
B. $3y^2-2x^2+5xy$
C. y^3+2x^2-7xy
D. y^3-2x^2+5xy

Exercise 10.17 The lengths of the base of a triangular prism are $2x^2+3y$, x^3-y^2, and $4y^2$, and its height is $5x^2-2y^3$.

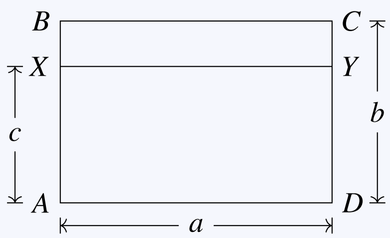
(a) Express the lateral surface area of the triangular prism in terms of x and y.
(b) Hence, find the lateral surface area of the prism when $x=2$ and $y=1$.

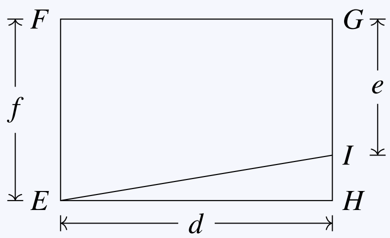
Exercise 10.18 Simplify the following algebraic expressions and arrange them in descending order of power of x.

(a) $2(3a+b)(4b-2a)$
(b) $5(x+2)(2x-3)$
(c) $3(5x^4+2y^2)(y-3x^2)$
(d) $(b^2+4)(-2b)-5b(3-b^2)$
(e) $(2x-3)^2-(4x+1)(2-x)$
(f) $(3p-1)^2-(2p+4)(p-3)$
(g) $(2a+5)(4a^2-a+1)-3(2a^2+1)$
(h) $(x^2-3x)(x+4)-(2x-1)^2$

Exercise 10.19 Below are the visualisation of expanding a polynomial.

1. In the left figure, $ABCD$ and $AXYD$ are rectangles. By considering the area of $XBCY$, show that $a(b-c)=ab-ac$. Hence, simplify $63(x-4)$.
2. In the right figure, $EFGH$ is a rectangle. By considering the area of $EFGI$, show that $d(e+f)=de+df$. Hence, simplify $65(x+8)$.
3. Hence, simplify $65(8+y)-63(y-4)$.





11. Angles related to Lines

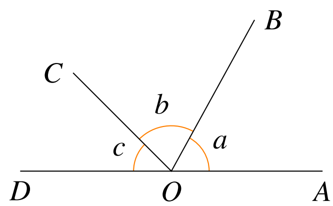
“The laws of nature are but the mathematical thoughts of God”

EUCLID

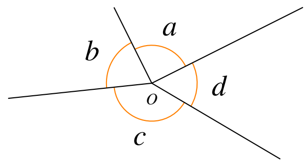
11.1 Angles Related to Intersecting Lines

When two or more lines intersect, there are three facts in Euclidean Geometry.

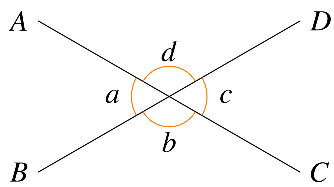
Definition 11.1 — Adjacent angles on straight line. (Abbreviation: *adj. ∠s on st. line*). If AOC is a *straight line*, then $a+b+c=180°$



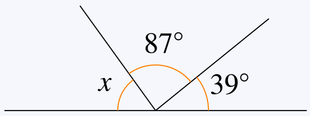
Definition 11.2 — Angles at a point. (Abbreviation: *∠s at a pt.*). If a,b,c and d are angles at the point O, then $a+b+c+d=360°$



Definition 11.3 — Vertically opposite angles. (Abbreviation: *vert. opp. ∠s*). If AC intersects BD at O, then $a=c$ and $b=d$

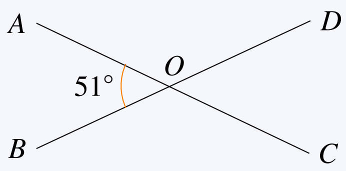


Exercise 11.1 In the figure, find x.



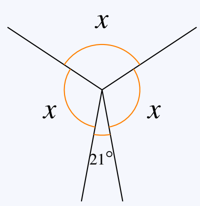
A. 45° B. 48° C. 54° D. 61°

Exercise 11.2 In the figure, AOB and COD are straight lines. $\angle AOD + \angle COB =$



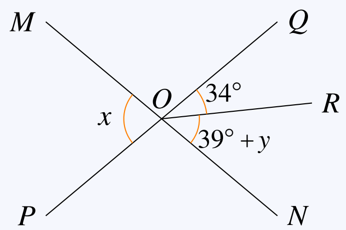
A. 102° B. 129° C. 204° D. 258°

Exercise 11.3 In the figure, find x.



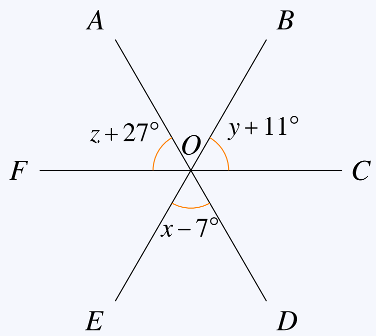
A. 110° B. 113° C. 116° D. 120°

Exercise 11.4 MON and POQ are straight lines. Find $x - y$.



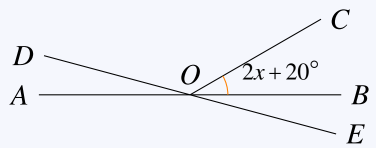
A. 51° B. 73° C. 107° D. 141°

Exercise 11.5 In the figure, AOD, BOE and COF are straight line. Find $x+y+z$.



A. 138° B. 142° C. 145° D. 149°

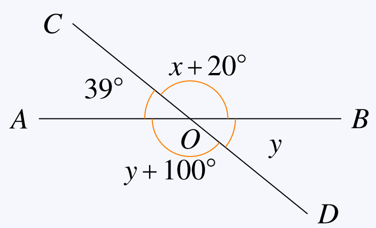
Exercise 11.6 In the figure, AOB and DOE are straight lines. Given that $\angle COB = 2\angle BOE$, express $\angle COD$ in terms of x.



A. $80° + 5x$ B. $90° + 4x$ C. $140° - 2x$ D. $150° - 3x$

Exercise 11.7 In the figure, AOB is a straight line. Which of the following statements is/are correct?

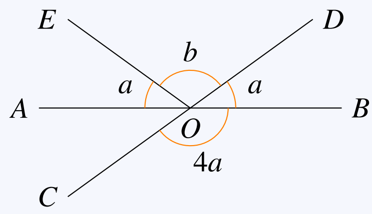
I. $x = 121°$
II. $y = 39°$
III. COD is a straight line.



A. I only
B. II only
C. I and III only
D. II and III only

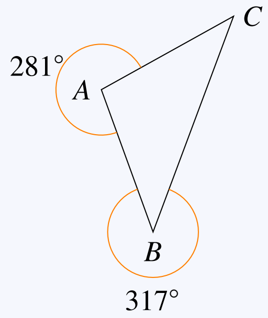
Exercise 11.8 In the figure, if AOB is a straight line, which of the following statements must be correct?

I. b and $2a$ are supplementary angles.
II. $\angle EOD$ and $\angle BOC$ are vertically opposite angles.
III. COD is a straight line.

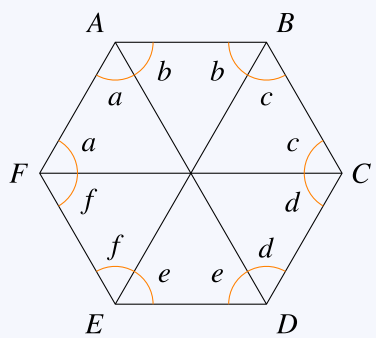


A. I only
B. II only
C. I and III only
D. II and III only

Exercise 11.9 In the figure, find $\angle ACB$.



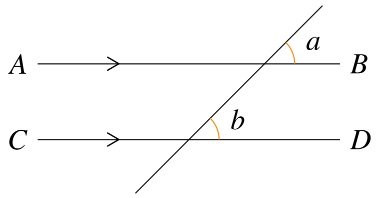
Exercise 11.10 In the figure, find $a+b+c+d+e+f$.



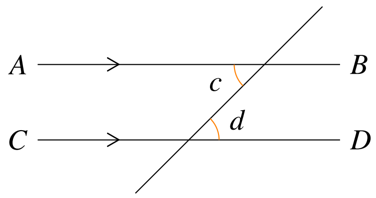
11.2 Angles Related to Parallel Lines

Similarly, we have three facts about parallel lines.

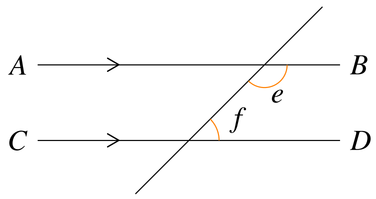
Definition 11.4 — Corresponding angles (Abbreviation: *corr.* ∠s, *AB // CD*). If $AB//CD$, then $a = b$



Definition 11.5 — Alternate angles (Abbreviation: *alt.* ∠s, *AB // CD*). If $AB//CD$, then $c = d$

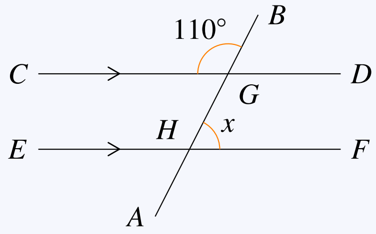


Definition 11.6 — Interior angles on the same side of the transversal (Abbreviation: *int.* ∠s, *AB // CD*). If $AB//CD$, then $e + f = 180°$



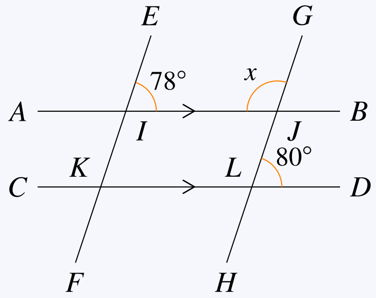
R If two angles are supplementary angles, their sum is 180°.

Exercise 11.11 In the figure, AB intersects CD and EF at G and H respectively. Then $x =$



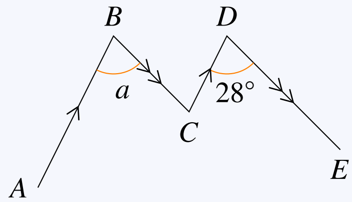
A. 54° B. 67° C. 96° D. 113°

Exercise 11.12 In the figure, $AIJB$, $CKLD$, $EIKF$ and $GJLH$ are straight lines. Find x.



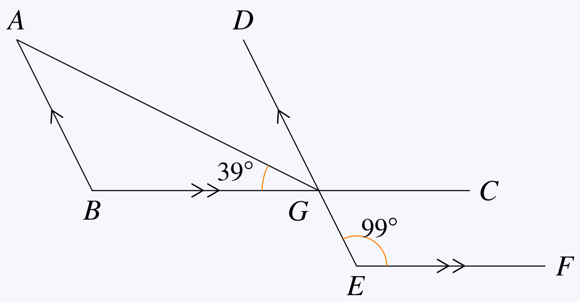
A. 78° B. 80° C. 100° D. 102°

Exercise 11.13 In the figure, find a.



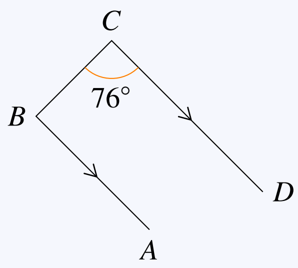
A. 21° B. 23° C. 27° D. 28°

Exercise 11.14 In the figure, BGC and DGC are straight lines. Find $\angle BAG$



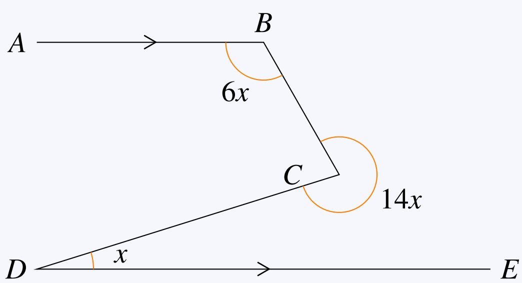
A. 42° B. 46° C. 51° D. 58°

Exercise 11.15 In the figure, find reflex $\angle ABC$.



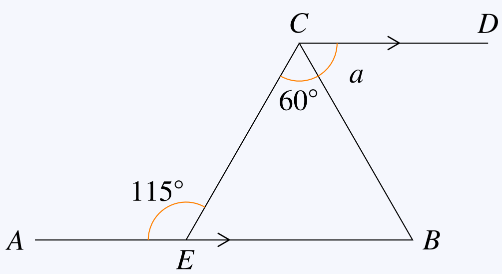
A. 76° B. 104° C. 194° D. 256°

Exercise 11.16 In the figure, find x

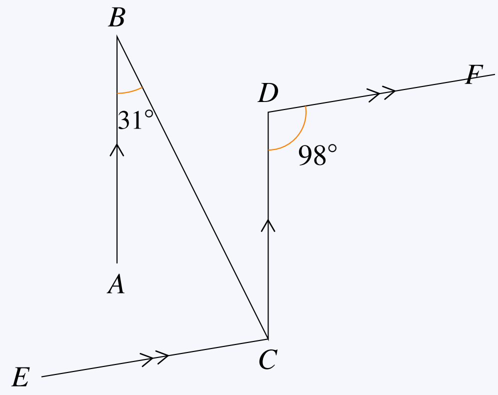


A. 18° B. 20° C. 22° D. 24°

Exercise 11.17 In the figure, AEB is a straight line. Find a.



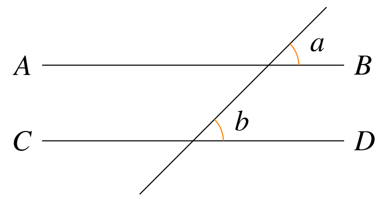
Exercise 11.18 Find $\angle BCE$.



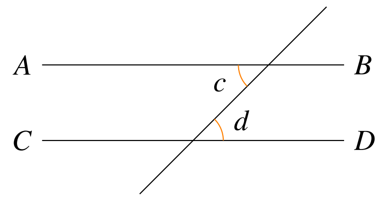
11.3 Identifying Parallel Lines

If we have two parallel lines, we have some relations between angles. Then will there be a reversed version? Indeed, fortunately, there are the inverse relations. In this situation we would call it as "*if and only if*" or in short *iff* or symbolically $\Longleftrightarrow$. For example, $AB//CD$ if and only if $a = b$ in the following first situation.

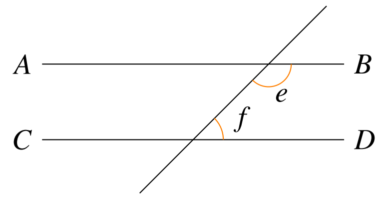
Definition 11.7 — The corresponding angles are equal. (Abbreviation: *corr. $\angle$s equal*). If $a = b$, then $AB//CD$.



Definition 11.8 — The alternate angles are equal. (Abbreviation: *alt. $\angle$s equal*). If $c = d$, then $AB//CD$.



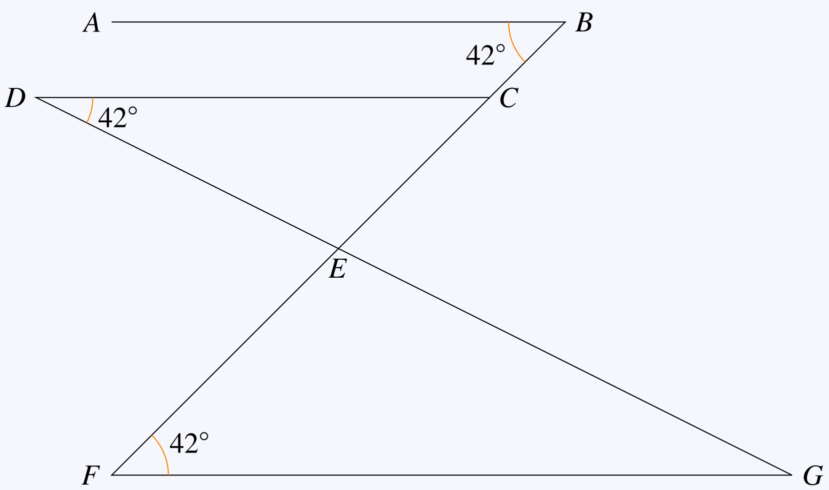
Definition 11.9 — The interior angles on the same side of the transversal are supplementary. (Abbreviation: *int. $\angle$s supp.*). If $e + f = 180°$, then $AB//CD$.



Exercise 11.19 Which of the following can be a reason for two straight lines being parallel.

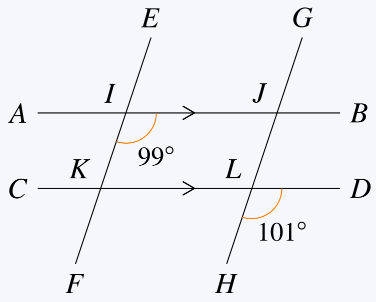
A. vert. opp. $\angle s$ equal
B. corr. $\angle s$ supp.
C. int. $\angle s$ supp.
D. alt. $\angle s$ supp.

Exercise 11.20 In the figure, DEG and $BCEF$ are straight lines. Which of the following must be correct?



A. $\angle EGF = 42°$
B. $CD//FG$
C. $AB//FG$
D. $AB//CD$

Exercise 11.21 In the figure, $EIKF$, $AIJB$ and $CKLD$ are straight lines. Which of the following statements must be correct?



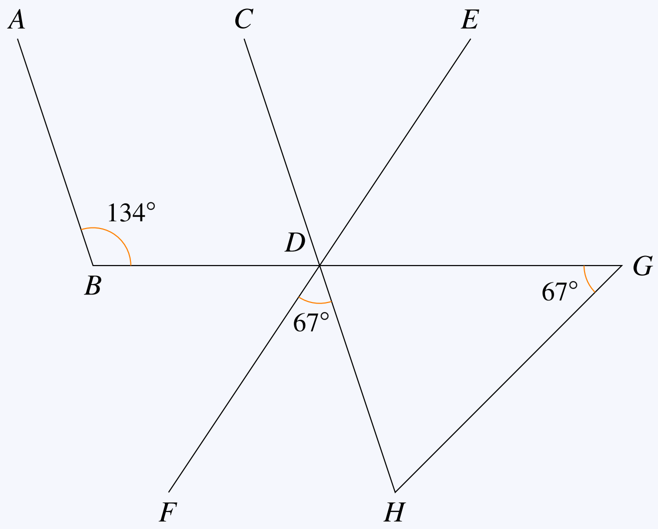
A. $\angle JLK = 101°$

B. $\angle IKL = 99°$

C. EF is not parallel to LH.

D. AI is not parallel to LD.

Exercise 11.22 In the figure, BDG and EDF are straight lines. If $EF//GH$, which of the following is correct?



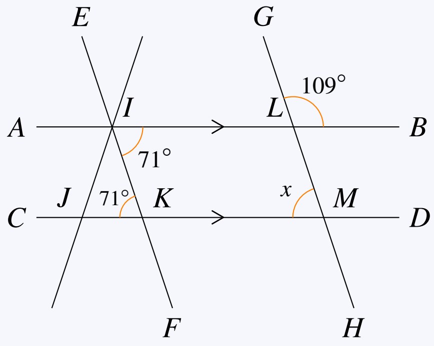
A. $\angle HDG = 40°$

B. $AB//DH$

C. CDH is a straight line if $\angle CDB = 67°$

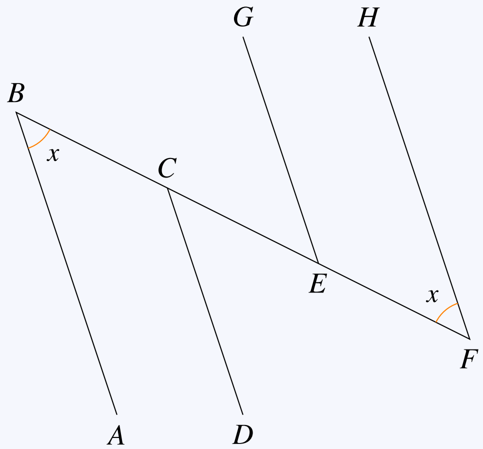
D. $\angle ABG$ and $\angle EDG$ are supplementary angles.

Exercise 11.23 In the figure, $AILB$, $CJKMD$, $EIKF$ and $GLMH$ are straight lines. Find x.

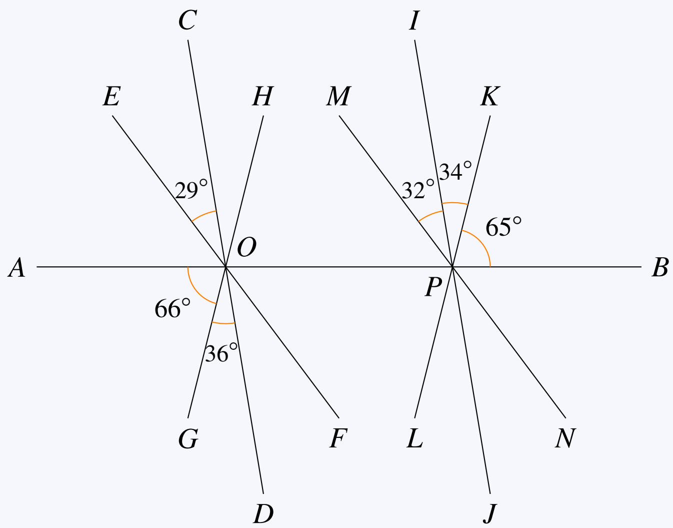


A. 71° B. 82° C. 96° D. 109°

Exercise 11.24 In the figure, $BCEF$ is a traight line. If $AB//CD$, is $CD//HF$? Explain briefly.



Exercise 11.25 In the figure, $AOPB$, COD, EOF, HOG, MPN, IPJ and KPL are straight lines. Prove $EF//MN$.



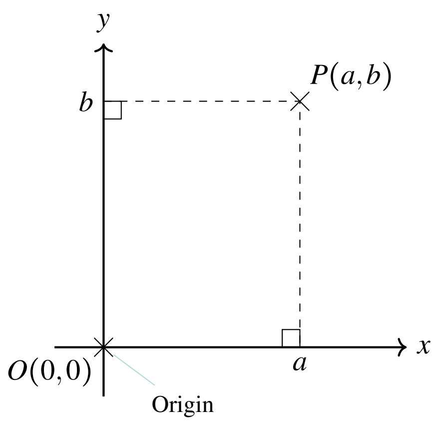
12. Introduction to Coordinates

"Cogito, ergo sum."

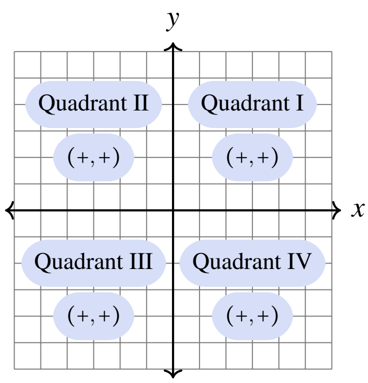
René Descartes

12.1 Rectangular Coordinate System

An ordered pair is a pair of numbers written as (a,b), which is used to describe the position of a point/an object on a coordinate plane. For any point lying on the x-axis, its y-coordinate is 0. For any point lying on the y-axis, its x-coordinate is 0.



Coordinate axes divide a rectangular coordinate plane into four quadrants.



Exercise 12.1 Which of the following points lies in quadrant II?

A. $(-6,-9)$ B. $(-8,7)$ C. $(4,4)$ D. $(5,-9)$

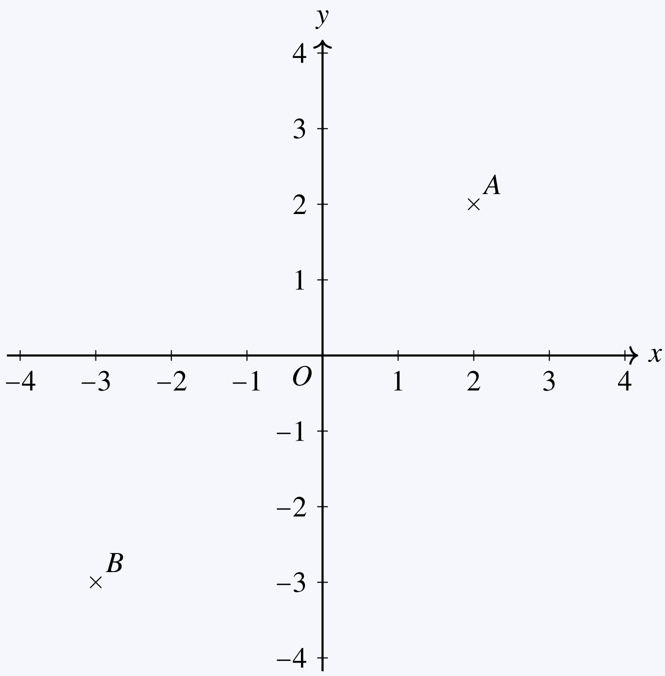
Exercise 12.2 Which of the following points lies on the x-axis?

A. $(0,7)$ B. $(1,2)$ C. $(9,0)$ D. $(-5,-9)$

Exercise 12.3 Given two points $P(a,b)$ and $Q(m,n)$ where P lies on the x-axis and Q lies on the y-axis. Which of the following must be correct?

A. $a+b=0$ B. $b=m$ C. $a+n=b+m$ D. $a=n$

Exercise 12.4 Referring to the figure, what is the x-coordinate of the intersection of the line AB and the y-axis?



A. 0 B. -1 C. 2 D. -3

Exercise 12.5 Consider a horizontal line in the rectangular coordinate system that contains the point $(7,-2)$. Which of the following statements about this line is/are correct?

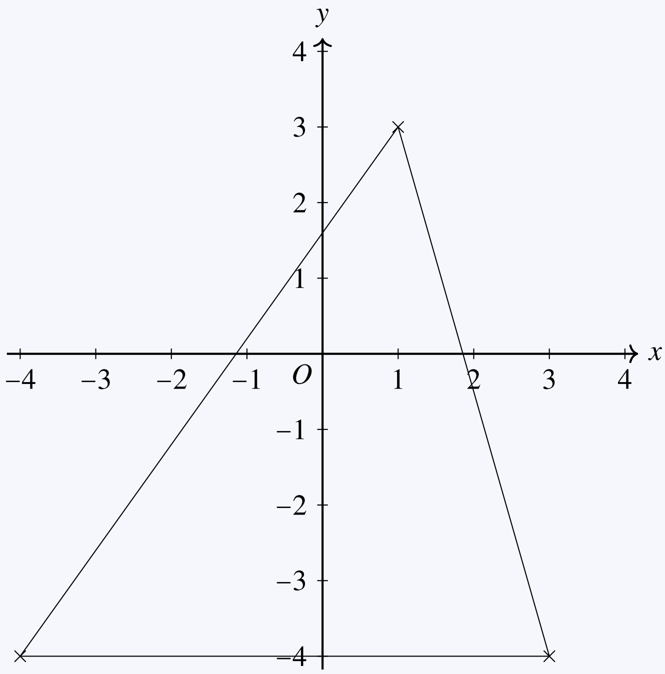
I. The y-coordinate of all the points on the line is -2.
II. The x-coordinate of all the points on the line is 7.
III. There are only three points lying on the line.

A. I only
B. II only
C. I and III only
D. II and III only

Exercise 12.6 $A(1,6)$, $B(1,-1)$, $D(8,6)$ and C are four vertices of a square. What are the coordinates of C?

A. $(-1,8)$ B. $(8,-1)$ C. $(1,-8)$ D. $(-8,1)$

Exercise 12.7 Which of the following points lies inside the triangle in the figure?



A. $(3,1)$ B. $(5,-2)$ C. $(-2,-3)$ D. $(1,-5)$

Exercise 12.8 AB is a line joining point $A(4,8)$ and $B(4,-1)$. Which of the following points lies on AB?

A. $(1,4)$ B. $(8,-1)$ C. $(-1,4)$ D. $(4,7)$

Exercise 12.9 M is the intersection of the vertical line containing the point $(1,7)$ and the horizontal line containing the point $(-3,-1)$. What is the coordinates of M?

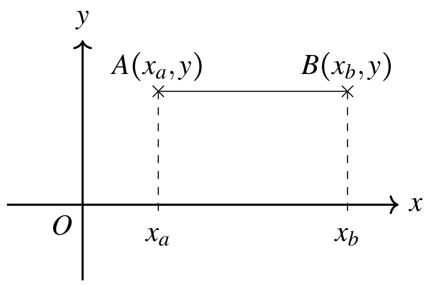
Exercise 12.10 Which quadrants do the following points lie in?

(a) $A(21,30)$
(b) $B(-4,-8)$
(c) $C(2.7,-4.6)$
(d) $D(-52.5,0.67)$

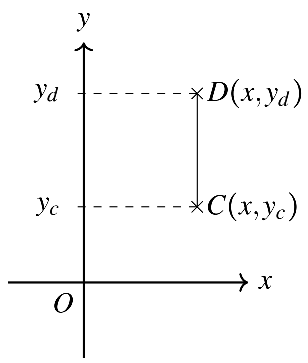
Exercise 12.11 The coordinates of A are $(-2,5)$. If B is a point lying on the x-axis and AB is parallel to the y-axis, what are the coordinates of B?

12.2 Distance between Two Points

Given that $A(x_a, y)$ and $B(x_b, y)$ lie on the same horizontal line. If $x_a < x_b$, then $AB = x_b - x_a$. Note that as A and B have the same y-coordinate, they lie on the same horizontal line.



Given that $C(x, y_c)$ and $D(x, y_d)$ lie on the same vertical line. If $y_c < y_d$, then $CD = y_d - y_c$. Note that as A and B have the same x-coordinate, they lie on the same vertical line.



Exercise 12.12 Find the distance between A(6, -1) and B(6, 5).

A. -6 units
B. 0 unit
C. 4 units
D. 6 units

Exercise 12.13 Given that $A(4, -2)$ and $B(b, -2)$ are two distinct points and $AB = 5$ units, which of the following is/are possible value(s) of b?

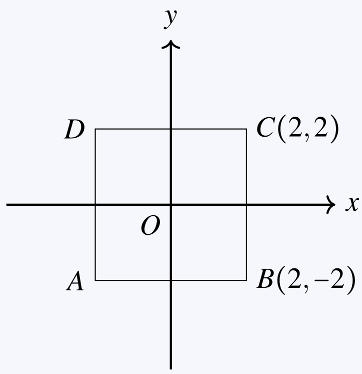
I. -7
II. 9
III. -1

A. I only
B. II only
C. I and III only
D. II and III only

Exercise 12.14 $A(1, 9)$, $B(5, 9)$, $C(1, 3)$ and $D(5, 3)$ are four points on the rectangular coordinate system. Which of the following statements are correct?

A. The line AB is a vertical line.
B. The line BD is a horizontal line.
C. $ABDC$ is a rectangle.
D. $ABDC$ is a square.

Exercise 12.15 In the figure, $ABCD$ is a square. What is the perimeter of the square?



A. 0 unit B. 4 units C. 8 units D. 16 units

Exercise 12.16 If $M(5,1)$, $N(5,9)$ and R are three distinct points and $MN = NR$, which of the following are possible coordinates of R?

A. $(5,5)$ B. $(-3,9)$ C. $(5,-4)$ D. $(-3,1)$

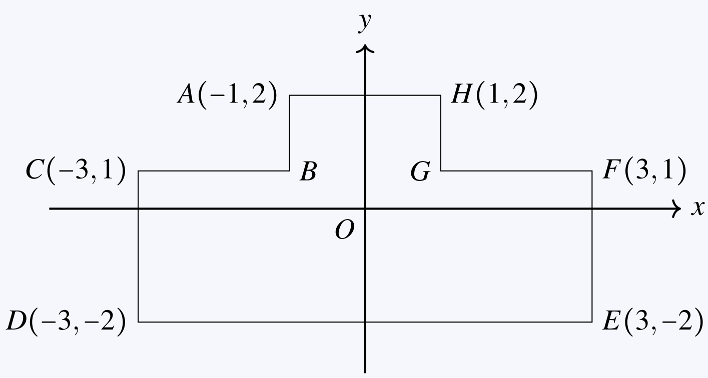
Exercise 12.17 Given that XY is a vertical line and the distance between $X(-9,h+3)$ and $Y(k-7,7)$ is 1 unit, which of the following is a possible value of $h+k$?

A. 4 B. 1 C. −3 D. −8

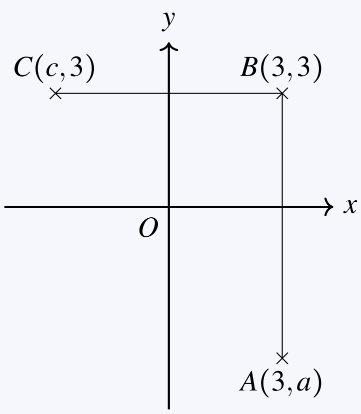
Exercise 12.18 B is on the right of $A(a,5)$ and directly above $C(3,c)$. What are the coordinates of B?

A. (c,a) B. $(3,5)$ C. $(5,3)$ D. (a,c)

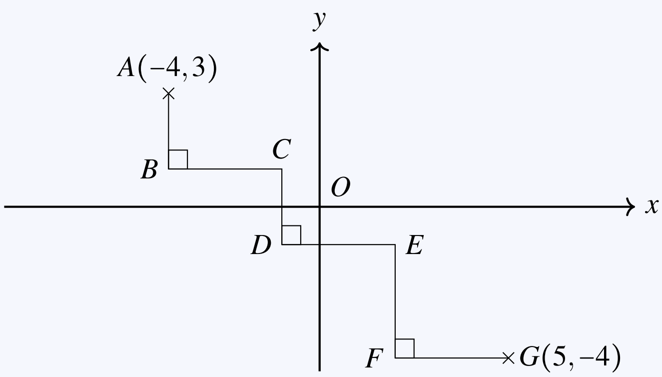
Exercise 12.19 Find the perimeter of $ABCDEFGH$.



Exercise 12.20 There are three points $A(3,a)$, $B(3,3)$ and $C(c,3)$ in the figure. If $AB = 7$ units and $BC = 6$ units, find a and c.



Exercise 12.21 In the figure, AB is parallel to the y-axis and FG is parallel to the x-axis. What is the length of the line $ABCDEFG$?

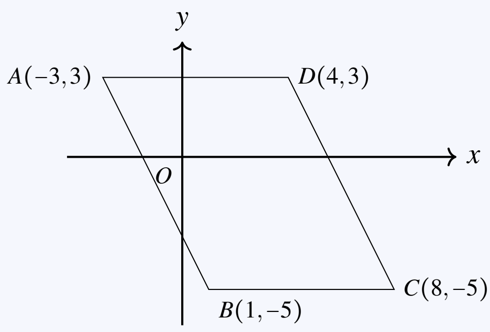


A. 7 units B. 9 units C. 12 units D. 16 units

12.3 Areas of Plane Figures

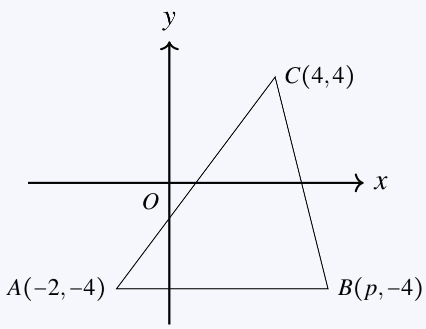
1. Use splitting method and filling method
2. Calculate the lengths of horizontal and vertical line segments

Exercise 12.22 Find the area of the parallelogram $ABCD$.



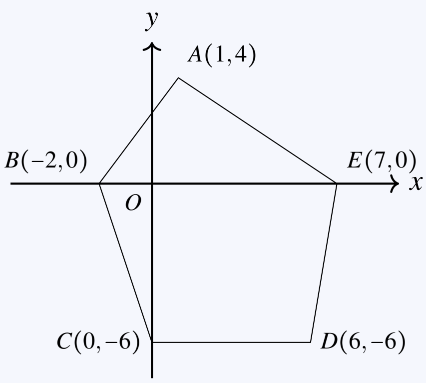
A. 14 sq. units B. 28 sq. units C. 42 sq. units D. 56 sq. units

Exercise 12.23 In the figure, the area of $\triangle ABC$ is 32 sq. units. Find p.



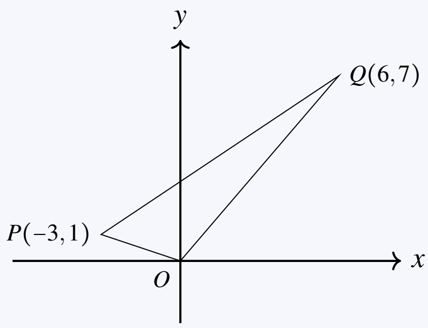
A. 4 B. 6 C. 8 D. 9

Exercise 12.24 In the figure, what is the area of pentagon $ABCDE$?



A. 48 sq. units B. 56 sq. units C. 63 sq. units D. 72 sq. units

Exercise 12.25 What is the area of the triangle in the figure?



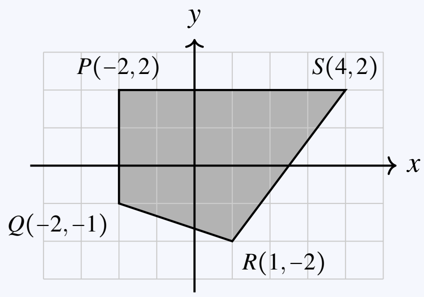
A. 13.5 sq. units
B. 14 sq. units
C. 14.5 sq. units
D. 15.5 sq. units

Exercise 12.26 $A(0,1)$, $B(0,-5)$ and C are three points on the rectangular coordinate plane. Given that the area of $\triangle ABC$ is 15 sq. units, which of the following are possible coordinates of C?

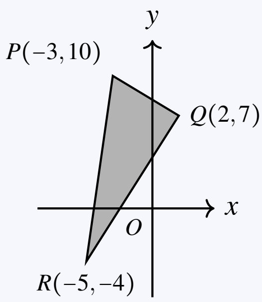
I. $(-5,0)$
II. $(5,-3)$
III. $(6,-2)$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

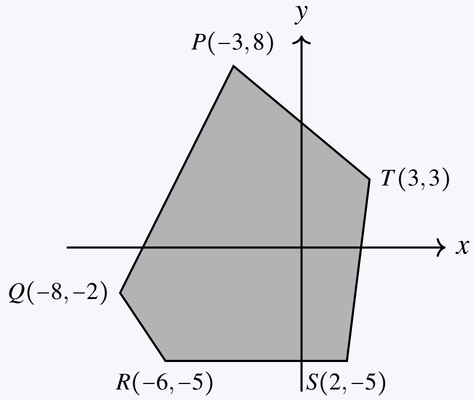
Exercise 12.27 Find the area of quadrilateral PQRS in the figure.



Exercise 12.28 Find the area of $\triangle ABC$ in the figure.



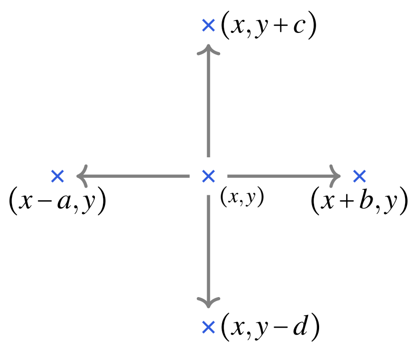
Exercise 12.29 Find the area of pentagon $PQRST$ in the figure.



12.4 Transformation of Points on the Coordinate Plate

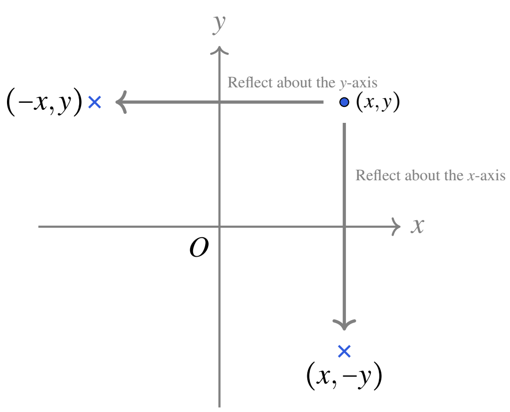
12.4.1 Translation

1. Translate to the *left* by a units: $(x-a, y)$
2. Translate to the *right* by b units: $(x+b, y)$
3. Translate *upwards* by c units: $(x, y+c)$
4. Translate *downwards* by d units: $(x, y-d)$



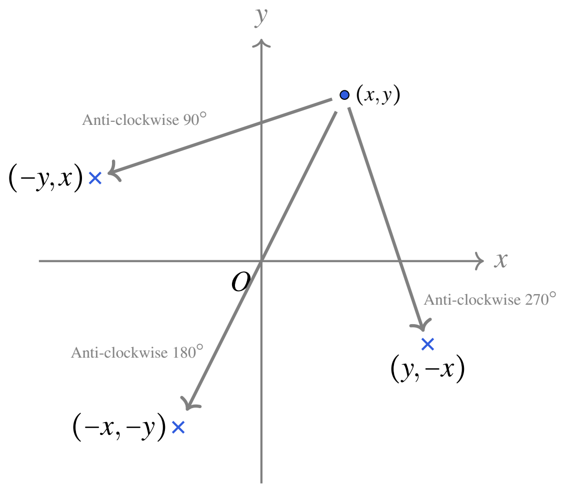
12.4.2 Reflection

1. Reflect *about the x-axis*: $(x, -y)$
2. Reflect *about the y-axis*: $(-x, y)$



12.4.3 Rotation

1. Rotate anti-clockwise about the origin through 90° or (Rotate clockwise about the origin through 270°): $(-y,x)$
2. Rotate anti-clockwise about the origin through 180° or (Rotate clockwise about the origin through 180°): $(-x,-y)$
3. Rotate anti-clockwise about the origin through 270° or (Rotate clockwise about the origin through 90°): $(y,-x)$



Exercise 12.30 $A(3,-5)$ is translated to the left by 3 units and then upwards by 2 units to A'. Find the coordinates of A'.

A. $(0,-7)$ B. $(0,-3)$ C. $(5,-3)$ D. $(6,-1)$

Exercise 12.31 If point B is translated to the left by 1 unit and then reflected about y-axis to $C(3,9)$. What are the coordinates of B?

A. $(0,9)$ B. $(-2,9)$ C. $(0,-9)$ D. $(-2,-9)$

Exercise 12.32 If point $A(4,3)$ is reflected about a vertical line containing the point $(1,6)$ and then translated downwards by 3 units to A', find the coordinates of A'.

A. $(-2,0)$ B. $(-2,6)$ C. $(-4,0)$ D. $(-4,6)$

Exercise 12.33 $A(5,8)$ undergoes some transformation to reach $B(-3,5)$. Which of the following is a possible procedure for the transformation?

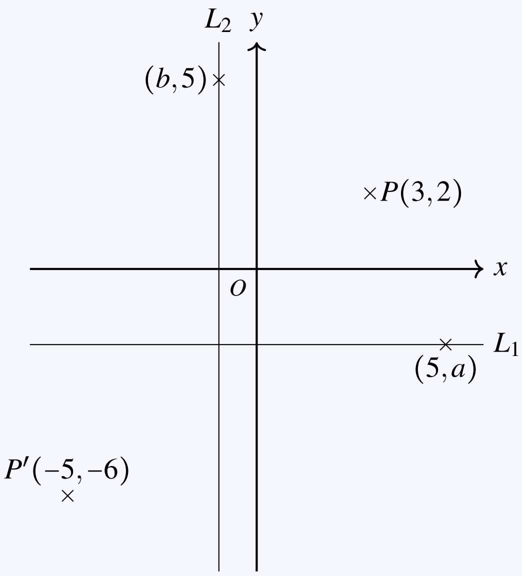
A. Rotate A anticlockwise about the origin through 90° and then translate it to the right by 5 units.
B. Translate A downwards by 5 units and then rotate it clockwise about the origin through 90°
C. Translate A to the right by 8 units and then tanslate upwards by 3 units.
D. Reflect A about a vertical line containing the point $(1,-8)$ and then translate it downwards by 2 units.

Exercise 12.34 $A(2,5)$ is rotated clockwise about the origin through $90°$ to A' and $B(-3,1)$ is translated downwards by 3 units to B'. What is the distance between A' and B'?

A. 5 units B. 6 units C. 8 units D. 11 units

Exercise 12.35 $M(2,3)$ is translated downwards by 7 units to M' and $N(-4,3)$ is rotated anticlockwise about the origin through $90°$ to N'. Find the area of trapezium $M'MNN'$.

Exercise 12.36 In the figure, L_1 is a horizontal line and L_2 is a vertical line. P is reflected about L_1 and then reflected about L_2 to P'. What is $a+b$?



Exercise 12.37 In the figure, A, B and C form a triangle. If A is rotated clockwise about the origin through $180°$ to A', find the difference between areas of $\triangle A'CB$ and $\triangle ABC$.