MATHEMATICS

Secondary II

NAME: ______________________________

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(VERSION: MAY 15, 2025)

Published by Publisher

First printing, May 2025

Contents

Book 2A

1. Errors in Measurement

1.1 Errors

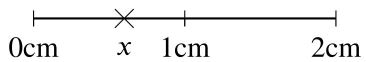
When we do approximation, there must be an error.

Definition 1.1 — Absolute error.
Absolute error is the absolute difference between the actual value and the approximation, which is always positive.

Then we may think, is there a limit for the absolute error? Or what does the absolute error depends on? Hence we have the concept of maximum absolute error.

Definition 1.2 — Maximum absolute error.
Maximum absolute error is the largest possible error of the measured value. It is half of the scale interval of the measuring tool.

Sometimes, we may not know what the actual value is, then we would use the concept of maximum absolute error to predict the "actual" absolute error, which must be less than or equal to the maximum absolute error. Normally in a scale, if the object is in between two interval, we can easily state the tendency of one to each side but not a finer division. Hence, the maximum absolute error is half of the scale interval. For example, we say the value x is around 1 cm, instead of 0 cm.



Then, when we have the maximum absolute error, we can talk about the range of the actual value, which is in between the upper limit and lower limit of the actual value.

$$\text{Upper limit of the actual value} = \text{Measured value} + \text{Maximum absolute error}$$
$$\text{Lower limit of the actual value} = \text{Measured value} - \text{Maximum absolute error}$$

R Aware the inequality sign in the range of the actual value:

$$\text{Lower limit of the actual value} \leq \text{Actual value} < \text{Upper limit of the actual value}$$

As the actual value can never equal to the upper limit of the actual value, using terms like "maximum" is not suitable. Hence, it is better to use "Upper limit" and "Lower limit", which are much more make sense and better.

To show the impact of the error, we have to compare the absolute error to the actual value. Hence we have the concept of relative error.

Definition 1.3 — Relative error.
Relative error is the measure of degree of accuracy of an approximation, which is the ratio of the absolute error to the actual value.

$$\text{Relative error} = \frac{\text{absolute error}}{\text{actual value}} \text{ OR } \frac{\text{maximum absolute error}}{\text{measured value}}$$

Definition 1.4 — Percentage error.
Percentage error is relative error $\times$ 100%.

Exercise 1.1 The weight of a box of candies is 785 g. If the relative error of the measurement is $\frac{1}{2000}$, find the maximum absolute error of this measurement. (Correct your answer to 3 significant figures.)

Exercise 1.2 The length of the equator of the earth is measured to be 12 756 km. If the percentage error is 0.1%, find the maximum absolute error of this measurement. (Correct your answer to 3 significant figures.)

Exercise 1.3 The distance between the earth and the moon is measured to be 385 000 km. If the percentage error is 2%, find the range of the true distance.

Exercise 1.4 Ayaka measured the weight of 50 \$1 coins and the measurement is 350 g, correct to the nearest g. (Correct your answer to 3 significant figures if necessary.)

(a) Find the percentage error of this measurement.

(b) If Ayaka uses the weight of 50 \$1 coins to estimate the weight of each \$1 coin, find the maximum absolute error of the weight of each \$1 coin.

(c) If the true weight of each \$1 coin is 7.02 g, using the result of (b), find the percentage error of the weight of each \$1 coin.

Exercise 1.5 The base and the height of a parrallelogram are 12.3 cm and 5.6 cm respectively.

(a) Find the upper limits and lower limits of the base and the height of the parallelogram respectively.

(b) Find the upper limit and lower limit of the area of the parallelogram. (Correct your answer to 3 significant figures.)

Exercise 1.6 The approximate values of x and y are 7.5 and 3.5 respectively, correct to 2 significant figures.

(a) Find the upper limits and lower limits of x and y respectively.

(b) FInd the upper limit and lower limit of $x + y$.

(c) Find the upper limit and lower limit of $x - y$.

1.2 DSE Corner

1.2.1 Long Questions

Problem 1.1 — 2013/I/Q8.

A pack of sea salt is termed *regular* if its weight is measured as 100 g correct to the nearest g.

(a) Find the least possible weight of a *regular* pack of sea salt.

(b) Is it possible that the total weight of 32 *regular* packs of sea salt is measured as 3.1 kg correct to the nearest 0.1 kg? Explain your answer.

Problem 1.2 — 2017/I/Q9.

A bottle is termed *standard* if its capacity is measured as 200 mL correct to the nearest 10 mL.

(a) Find the least possible capacity of a *standard* bottle.

(b) Someone claims that the total capacity of 120 *standard* bottles can be measured as 23.3 L correct to the nearest 0.1 L. Do you agree? Explain your answer.

Problem 1.3 — 2023/I/Q3.

A packet of cheese is termed *regular* if its weight is measured as 220 g correct to the nearest 10 g. Someone claims that the total weight of 250 *regular* packets of cheese can be measured as 53.6 kg correct to the nearest 0.1 kg. Is this claim correct? Explain your answer.

1.2.2 Multiple Choice Questions

Problem 1.4 — SP/II/Q15.

The lengths of the three sides of a triangle are measured as 15 cm, 24 cm and 25 cm respectively. If the three measurements are correct to the nearest cm, find the percentage error in calculating the perimeter of the triangle correct to the nearest 0.1%.

A. 0.8% B. 2.3% C. 4.7% D. 6.3%

Problem 1.5 — 2012/II/Q14.

The length of a piece of thin string is measured as 25 m correct to the nearest m. If the string is cut into n pieces such that the length of each piece is measured as 5 cm correct to the nearest cm, find the greatest possible value of n.

A. 445 B. 566 C. 567 D. 650

Problem 1.6 — 2014/II/Q11.

The width and the length of a thin rectangular metal sheet are measured as 8 cm and 10 cm correct to the nearest cm respectively. Let x cm^2 be the actual area of the metal sheet. Find the range of values of x.

A. $71.25 \le x < 89.25$

B. $71.25 < x \le 89.25$

C. $79.5 \le x < 80.5$

D. $79.5 < x \le 80.5$

Problem 1.7 — 2015/II/Q14.

There is a bag of white sugar. The weight of white sugar in the bag is measured as 5 kg correct to the nearest kg. If the bag of white sugar is packed into n packets such that the weight of white sugar in each packet is measured as 10 g correct to the nearest g, find the greatest possible value of n.

A. 429 B. 500 C. 578 D. 579

Problem 1.8 — 2021/II/Q5.

If $x = 6.24$ (correct to 2 decimal places), find the range of values of x.

A. $6.23 < x \leq 6.25$
B. $6.23 \leq x < 6.25$
C. $6.235 < x \leq 6.245$
D. $6.235 \leq x < 6.245$

Problem 1.9 — 2022/II/Q6.

It is given that x is a real number. If x is rounded down to 3 significant figures, then the result is 345. Find the range of values of x.

A. $344 < x \leq 345$
B. $345 \leq x < 346$
C. $345 < x \leq 345.5$
D. $344.5 \leq x < 345.5$

Problem 1.10 — 2023/II/Q7.

If $y = 73.8$ (correct to 3 significant figures), find the range of values of y.

A. $73.7 < y \leq 73.9$
B. $73.7 \leq y < 73.9$
C. $73.75 < y \leq 73.85$
D. $73.75 \leq y < 73.85$

2. Identities and Factorisation

2.1 Identities

An *identity* is an equation that can be satisfied by ALL values of the unknown(s). We would use $\equiv$ instead of $=$ to represent an identity.

To find some unknown constants in an identity, we can expand and simplify both sides of an equation individually, and then compare the terms on both sides to obtain the values of the unknown(s).

There are some important identities:

$$a^2 - b^2 \equiv (a+b)(a-b)$$
$$(a \pm b)^2 \equiv a^2 \pm 2ab + b^2$$

2.2 Factorisation

The process of expressing an algebraic expression as a product of its factors is called *factorization*. Factorisation is the reverse process of *expansion*. It is useful when we have to simplify some algebraic fractions or expression. There are some skills to factorise:

(a) Common factors (b) Grouping terms (c) Identities

Exercise 2.1 (a) If $x^2 + y^2 = 50$ and $xy = 13$, find the values of $(x+y)^2$ and $(x-y)^2$.
(b) If $(x+y)^2 = 60$ and $(x-y)^2 = 20$, find the values of xy and $x+y$.

Exercise 2.2 Factorise the following expressions.

1. $-12ac-24cd$
2. $-12x^2c+3x^2$
3. $m(m+n)+(m+n)$
4. $x(x-y)+y(y-x)$
5. $ec+ed-fc-fd$
6. $4c^2-2c+2cd-d$
7. $10xz-15xy-8yz+12y^2$
8. $pqs-3at-pqt+3as$
9. $9t^2-16$
10. $121x^2-225y^2$
11. $b^2+8b+16$
12. $26b-169-b^2$
13. $25p^2-70pq+49q^2$
14. $64x^2+48xy+9y^2$
15. $x^2y^4-16x^2$
16. $18a^2+15b-50b^2-9a$

Exercise 2.3 Expand $(x+1)(x-1)(x^2-1)$. Hence, find the value of $11\times9\times99$.

Exercise 2.4 Expand $(x+y+z)^2$ by using $(x+a)^2=x^2+2ax+a^2$.

Exercise 2.5 Expand $(x+y-z)(x+y+z)$ by using $(a+b)(a-b)=a^2-b^2$.

Exercise 2.6 (a) Factorise $4x^2+12xy+9y^2$ and $4x^2-12xy+9y^2$.
(b) Hence factorise $(4x^2+12xy+9y^2)(4x^2-12xy+9y^2)-9x^4$.

Exercise 2.7 Factorise $x^2-12x+36$ and $25y^2+40y+16$. Hence factorise $x^2-25y^2-12x-40y+20$.

Exercise 2.8 Factorise $3p^2+6p+3$. Hence factorise $3\alpha^2+6\alpha-72$.

2.3 DSE Corner

2.3.1 Long Questions

Problem 2.1 — PP/I/Q3.
Factorise

(a) $9x^2-42xy+49y^2$,

(b) $9x^2-42xy+49y^2-6x+14y$.

Problem 2.2 — 2012/I/Q3.
Factorise

(a) $x^2-6xy+9y^2$,

(b) $x^2-6xy+9y^2+7x-21y$.

Problem 2.3 — 2013/I/Q3.
Factorise

(a) $4m^2-25n^2$,

(b) $4m^2-25n^2+6m-15n$.

Problem 2.4 — 2015/I/Q4.
Factorise

(a) $x^3+x^2y-7x^2$,

(b) $x^3+x^2y-7x^2-x-y+7$.

Problem 2.5 — 2018/I/Q5.
Factorise

(a) $9r^3 - 18r^2s$,

(b) $9r^3 - 18r^2s - rs^2 + 2s^3$.

Problem 2.6 — 2022/I/Q4.
Factorise

(a) $9c^2 - 6c + 1$,

(b) $(4c - d)^2 - 9c^2 + 6c - 1$.

2.3.2 Multiple Choice Questions

Problem 2.7 — SP/II/Q3.
$a^2 - b^2 + 2b - 1 =$

A. $(a-b-1)(a+b-1)$.
B. $(a-b-1)(a+b+1)$.
C. $(a-b+1)(a+b-1)$.
D. $(a-b+1)(a-b-1)$.

Problem 2.8 — PP/II/Q1.
$x^3(2x+x) =$

A. $3x^4$.
B. $2x^5$.
C. $3x^5$.
D. $2x^6$.

Problem 2.9 — PP/II/Q3.
$p^2 - q^2 - p - q =$

A. $(p+q)(p-q-1)$.
B. $(p+q)(p+q-1)$.
C. $(p-q)(p-q+1)$.
D. $(p-q)(p+q-1)$.

Problem 2.10 — 2012/II/Q2.
$(4x+y)^2 - (4x-y)^2 =$

A. 0.
B. $2y^2$.
C. $8xy$.
D. $16xy$.

Problem 2.11 — 2013/II/Q3.
$hl - kl + hm - km - hn + kn =$

A. $(h+k)(l-m+n)$.
B. $(h+k)(l+m-n)$.
C. $(h-k)(l-m+n)$.
D. $(h-k)(l+m-n)$.

Problem 2.12 — 2014/II/Q2.
$u^2 - v^2 - 5u + 5v =$

A. $(u-v)(u+v-5)$.
B. $(u-v)(u+v+5)$.
C. $(u+v)(u-v-5)$.
D. $(u+v)(u-v+5)$.

Problem 2.13 — 2016/II/Q3.
$16 - (2x-3y)^2 =$

A. $(4-2x-3y)(4+2x+3y)$.
B. $(4-2x-3y)(4+2x-3y)$.
C. $(4-2x+3y)(4+2x+3y)$.
D. $(4-2x+3y)(4+2x-3y)$.

Problem 2.14 — 2018/II/Q3.
$h^2 - 6h - 4k^2 - 12k =$

A. $(h-2k)(h-2k+6)$.
B. $(h-2k)(h+2k+6)$.
C. $(h+2k)(h-2k-6)$.
D. $(h+2k)(h+2k-6)$.

Problem 2.15 — 2020/II/Q4.
$(3a+2b)(4a-5b)-a(6a+4b)=$

A. $(3a+2b)(2a-5b)$.
B. $(3a+2b)(6a-5b)$.
C. $(3a-2b)(2a+5b)$.
D. $(3a-2b)(6a+5b)$.

Problem 2.16 — 2022/II/Q1.
$\alpha^2-\alpha-\beta^2+\beta=$

A. $(\alpha+\beta)(\alpha-\beta+1)$.
B. $(\alpha+\beta)(\alpha-\beta-1)$.
C. $(\alpha-\beta)(\alpha+\beta+1)$.
D. $(\alpha-\beta)(\alpha+\beta-1)$.

3. Algebraic Fractions and Formulae

3.1 Calculation of Algebraic fractions

Algebraic fraction is an algebraic expression in which two polynomials, A and B forming an expression $\frac{A}{B}$, and B is not a constant number.

Similar to numerical fractions, algebraic fractions can be reduced to its simpliest form by cancelling out the common factor(s) of its numerator and denominator. All manipulation of algebraic fractions are similar to numerical fractions.

3.2 Formulae

A formula is an equality relating two or more variables. Then we can find the value of a variable in a formula by substituting all other variables. If a variable is expressed in terms of other variables, it is called the subject of the formula. Many DSE questions will ask you to change the subject of a formula.

3.3 DSE Corner

3.3.1 Long Questions

Simplfy the following expressions and express your answer with positive indices.

Problem 3.1 — SP/I/Q1.

$$\frac{(xy)^2}{x^5y^6}$$

Problem 3.2 — PP/I/Q1.

$$\frac{(m^5n^{-2})^6}{m^4n^{-3}}$$

Problem 3.3 — 2012/I/Q1.

$$\frac{m^{-12}n^8}{n^3}$$

Problem 3.4 — 2013/I/Q1.

$$\frac{x^{20}y^{13}}{(x^5y)^6}$$

Problem 3.5 — 2014/I/Q1.

$$\frac{(xy^{-2})^3}{y^4}$$

Problem 3.6 — 2015/I/Q1.

$$\frac{m^9}{(m^3n^{-7})^5}$$

Problem 3.7 — 2016/I/Q1.

$$\frac{(x^8y^7)^2}{x^5y^{-6}}$$

Problem 3.8 — 2017/I/Q2.

$$\frac{(m^4n^{-1})}{(m^{-2})^5}$$

Problem 3.9 — 2018/I/Q2.

$$\frac{xy^7}{(x^{-2}y^3)^4}$$

Problem 3.10 — 2020/I/Q1.

$$\frac{(mn^{-2})^5}{m^{-4}}$$

Problem 3.11 — 2021/I/Q1.

$$(\alpha\beta^3)(\alpha^{-2}\beta^4)^5$$

Problem 3.12 — 2022/I/Q1.

$$\frac{(a^3b^{-2})^4}{a^{-5}b^6}$$

Problem 3.13 — 2023/I/Q2.

$$\frac{x^{-8}y}{(x^7y^9)^{-6}}$$

Problem 3.14 — SP/I/Q2.

Make b the subject of the formula $a(b+7) = a+b$.

Problem 3.15 — PP/I/Q2.

Make a the subject of the formula $\dfrac{5+b}{1-a} = 3b$.

Problem 3.16 — 2012/I/Q2.

Make a the subject of the formula $\dfrac{3a+b}{8} = b-1$.

Problem 3.17 — 2013/I/Q2.

Make k the subject of the formula $\dfrac{3}{h} - \dfrac{1}{k} = 2$.

Problem 3.18 — 2014/I/Q5.
Consider the formula $2(3m+n)=m+7$.
(a) Make n the subject of the above formula.
(b) If the value of m is increased by 2, write down the change in the value of n.

Problem 3.19 — 2015/I/Q2.
Make b the subject of the formula $\frac{4a+5b-7}{b}=8$.

Problem 3.20 — 2016/I/Q2.
Make x the subject of the formula $Ax=(4x+B)C$.

Problem 3.21 — 2017/I/Q1.
Make y the subject of the formula $k=\frac{3x-y}{y}$.

Problem 3.22 — 2018/I/Q1.
Make b the subject of the formula $\frac{a+4}{3}=\frac{b+1}{2}$.

Problem 3.23 — 2019/I/Q1.
Make h the subject of the formula $9(h+6k)=7h+8$.

Problem 3.24 — 2021/I/Q2.
Make a the subject of the formula $\frac{4-3a}{b}=5$.

Problem 3.25 — 2022/I/Q1.
Simplfy $\frac{3}{k-9}+\frac{2}{5k+6}$.

Problem 3.26 — 2023/I/Q1.
Make h the subject of the formula $\frac{5}{h+k}=\frac{k}{h-3}$.

3.3.2 Multiple Choice Questions

Problem 3.27 — SP/II/Q31.
$\frac{1}{2-x}+\frac{x-1}{(x-2)^2}=$

A. $\frac{-3}{(2-x)^2}$. B. $\frac{1}{(2-x)^2}$. C. $\frac{-2x+3}{(2-x)^2}$. D. $\frac{2x-3}{(2-x)^2}$.

Problem 3.28 — 2015/II/Q31.
$\frac{1}{x^2-2x+1}-\frac{1}{x^2+x-2}=$

A. $\frac{1}{(x-1)(x+2)}$.
B. $\frac{1}{(x-1)^2(x+2)}$.
C. $\frac{3}{(x-1)^2(x+2)}$.
D. $\frac{2x+1}{(x-1)^2(x+2)}$.

Problem 3.29 — 2018/II/Q4.
$\frac{1}{3x+7}-\frac{1}{3x-7}=$

A. $\frac{14}{49-9x^2}$. B. $\frac{14}{9x^2-49}$. C. $\frac{6x}{49-9x^2}$. D. $\frac{6x}{9x^2-49}$.

Problem 3.30 — 2020/II/Q3.
$\frac{5}{4k+3} - \frac{2}{4k-3} =$

A. $\frac{12k-21}{16k^2-9}$. B. $\frac{12k+9}{16k^2-9}$. C. $\frac{14k-21}{16k^2-9}$. D. $\frac{14k+9}{16k^2-9}$.

Problem 3.31 — 2021/II/Q4.
$\frac{6}{n-6} - \frac{7}{n-7} =$

A. $\frac{n}{(n-6)(n-7)}$.
B. $\frac{n}{(n-6)(7-n)}$.
C. $\frac{n+84}{(n-6)(n-7)}$.
D. $\frac{n+84}{(n-6)(7-n)}$.

Problem 3.32 — 2023/II/Q2.
$\frac{2}{5-4x} - \frac{1}{5+4x} =$

A. $\frac{5+4x}{25-16x^2}$. B. $\frac{5-4x}{25-16x^2}$. C. $\frac{5+12x}{25-16x^2}$. D. $\frac{5-12x}{25-16x^2}$.

Problem 3.33 — SP/II/Q2.
If $5-3m = 2n$, then $m =$

A. n. B. $\frac{2n-5}{3}$. C. $\frac{-2n+5}{3}$. D. $\frac{-2n+15}{3}$.

Problem 3.34 — PP/II/Q2.
If $3a+1 = 3(b-2)$, then $b =$

A. $a+1$. B. $a+3$. C. $a+\frac{7}{3}$. D. $a-\frac{5}{3}$.

Problem 3.35 — 2013/II/Q2.
If $\frac{y-1}{c} = \frac{y+1}{c}$, then $y =$

A. $\frac{c-d}{c+d}$. B. $\frac{d-c}{c+d}$. C. $\frac{c+d}{c-d}$. D. $\frac{c+d}{d-c}$.

Problem 3.36 — 2016/II/Q2.
If $\frac{a}{x} + \frac{b}{y}$, then $x =$

A. $\frac{ay}{3y-b}$. B. $\frac{ay}{b-3y}$. C. $\frac{by}{3y-a}$. D. $\frac{by}{a-3y}$.

Problem 3.37 — 2017/II/Q3.
If $\frac{a+4b}{2a} = 2+\frac{b}{a}$, then $a =$

A. $\frac{2b}{3}$. B. $\frac{3b}{2}$. C. $\frac{5b}{6}$. D. $\frac{6b}{5}$.

Problem 3.38 — 2018/II/Q2.
If $\frac{\alpha}{1-x} = \frac{\beta}{x}$, then $x =$

A. $\frac{\alpha}{\alpha-\beta}$. B. $\frac{\alpha}{\alpha+\beta}$. C. $\frac{\beta}{\alpha-\beta}$. D. $\frac{\beta}{\alpha+\beta}$.

Problem 3.39 — 2019/II/Q5.
If $h=3-\frac{5}{k+4}$, then $k=$

A. $\frac{4h-7}{3-h}$.
B. $\frac{4h-17}{3-h}$.
C. $\frac{4h-7}{3+h}$.
D. $\frac{4h-17}{3+h}$.

Problem 3.40 — 2020/II/Q2.
If $a(a+b)=2(b-a)$, then $b=$

A. $\frac{a^2+a}{2+a}$.
B. $\frac{a^2-2a}{2+a}$.
C. $\frac{a^2+2a}{2+a}$.
D. $\frac{a^2-a}{2+a}$.

Problem 3.41 — 2021/II/Q2.
If $m(m-a)=a(1-m)$, then $a=$

A. m.
B. $2m$.
C. m^2.
D. $\frac{m^2+m}{2}$.

Problem 3.42 — 2022/II/Q3.
If $\frac{2}{u}+\frac{3}{v}=4$, then $u=$

A. $\frac{2v}{4v-3}$.
B. $\frac{2v}{3-4v}$.
C. $\frac{3v}{4v-2}$.
D. $\frac{3v}{2-4v}$.

Problem 3.43 — 2023/II/Q1.
If $\frac{a+5b}{7a+2b}=\frac{1}{b+3}$, then $a=$

A. $\frac{4-b}{5b^2+13b}$.
B. $\frac{4+b}{5b^2+13b}$.
C. $\frac{5b^2+13b}{4-b}$.
D. $\frac{5b^2+13b}{4+b}$.

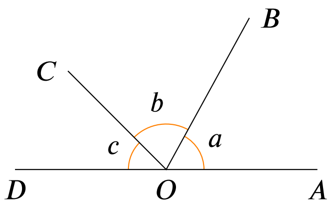
4. Angles related to Rectilinear Figures

4.1 Angles related to Lines

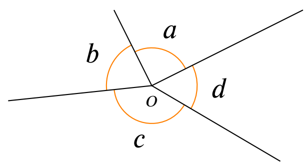
We had learnt angles related to intersecting lines and parallel lines in Form 1 Chapter 11. The following is the revision.

4.1.1 Angles related Intersecting Lines

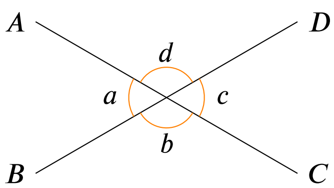
Definition 4.1 — Adjacent angles on a straight line (Abbreviation: *adj. ∠s on st. line*).
If AOC is a *straight line*, then $a+b+c=180°$.



Definition 4.2 — Angles at a point (Abbreviation: *∠s at a pt.*).
If a, b, c and d are angles at the point O, then $a+b+c+d=360°$.

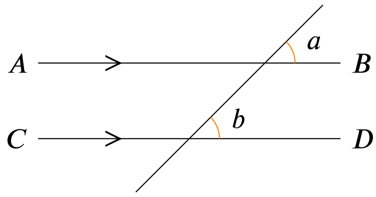


Definition 4.3 — Vertically opposite angles (Abbreviation: *vert. opp. ∠s*).
If AC intersects BD at O, then $a=c$ and $b=d$.

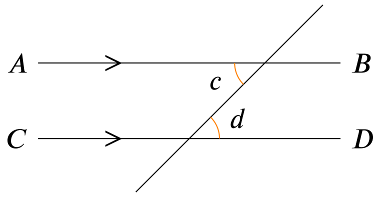


4.1.2 Angles related Parallel Lines

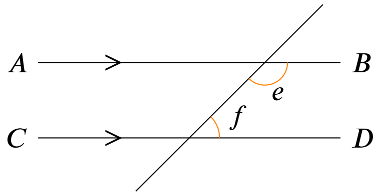
Definition 4.4 — Corresponding angles (Abbreviation: *corr. ∠s, AB // CD*).
If $AB//CD$, then $a = b$.



Definition 4.5 — Alternate angles (Abbreviation: *alt. ∠s, AB // CD*).
If $AB//CD$, then $c = d$.



Definition 4.6 — Interior angles on the same side of the transversal (Abbreviation: *int. ∠s, AB // CD*).
If $AB//CD$, then $e + f = 180°$.

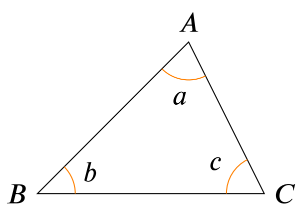


4.2 Angles related to Triangles

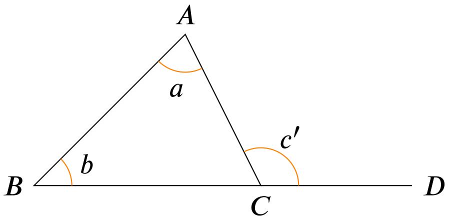
4.2.1 Angles of a triangle

We had learnt ∠ sum of △ in Form 1 Chapter 5. Now we add one more property.

Definition 4.7 — Angle sum of triangle (Abbreviation: *∠ sum of △*).
In $\triangle ABC$, $\angle A + \angle B + \angle C = 180°$.



Definition 4.8 — Exterior angle of a triangle (Abbreviation: *ext. ∠ of △*).
In $\triangle ABC$, $c' = a + b$.



4.2.2 Triangle properties

Definition 4.9 — Base angles of an isosceles triangle are the same (Abbreviation: *base ∠s, isos. △*).
In $\triangle ABC$, if $AB = AC$, then $\angle B = \angle C$.

Definition 4.10 — Sides opposite equal angles (Abbreviation: *sides opp. equal ∠s*).
In $\triangle ABC$, if $\angle B = \angle C$, then $AB = AC$.

Definition 4.11 — Properties of Isosceles Triangles (Abbreviation: *base ∠s, isos. △*).
For an isosceles triangle ABC with $AB = AC$, if one of the following conditions is true, then the other two conditions are also true.

1. $BD = CD$
2. $\angle BAD = \angle CAD$
3. $AD \perp BC$

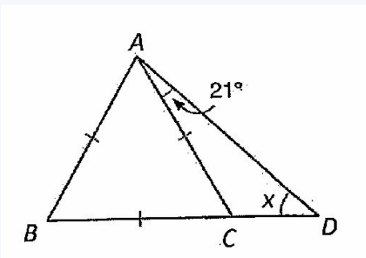
Definition 4.12 — Properties of Equilateral Triangles (Abbreviation: *prop. of equil. △*).
In $\triangle ABC$,
1. if $AB = BC = AC$, then $\angle A = \angle B = \angle C = 60°$.
2. if $\angle A = \angle B = \angle C = 60°$, then $\triangle ABC$ is an equilateral triangle.

4.3 Angles related to Polygon

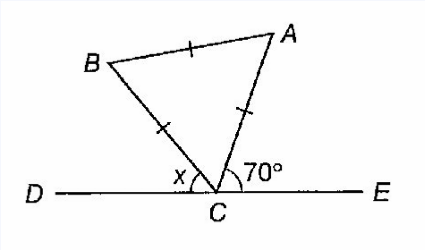
Definition 4.13 — Sum of interior angles of a polygon (Abbreviation: *∠ sum of polygon*).
The sum of all the interior angles of an n-sided polygon is $(n-2) \times 180°$.

Definition 4.14 — Sum of exterior angle of a polygon (Abbreviation: *ext. ∠s of polygon*).
The sum of all the exterior angles of a convex polygon is 360°.

Exercise 4.1 In the figure, BCD is a straight line and $AB = BC = AC$. Find x.



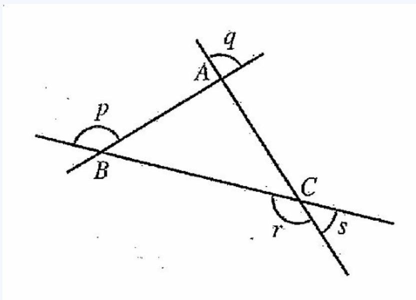
Exercise 4.2 In the figure, DCE is a straight line, $AB = BC = AC$ and $\angle ACE = 70°$. Find x.



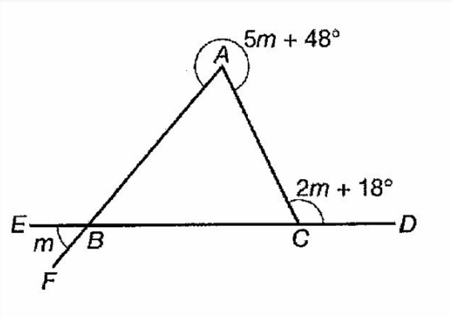
Exercise 4.3 Which of the following about the figure are NOT true?

I. $p+q+r = 360°$ II. $q+s = r$ III. $AB = AC$

A. I and II only
B. I and III only
C. II and III only
D. I, II and III

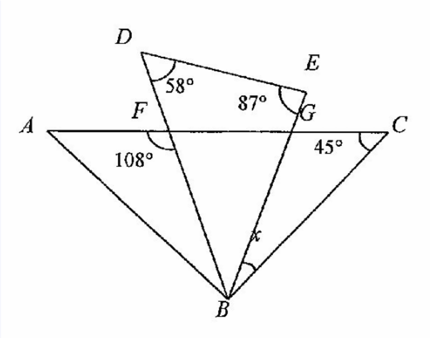


Exercise 4.4 In the figure, ABF and $EBCD$ are straight lines. Find $\angle BAC$.



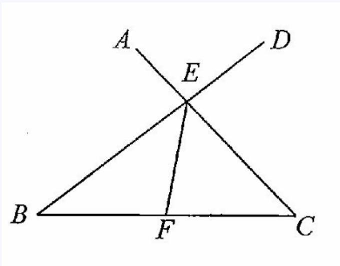
A. 49° B. 57° C. 63° D. 67°

Exercise 4.5 In the figure, AC cuts BD and BE at point F and G respectively. Find x.



A. 28° B. 35° C. 42° D. 63°

Exercise 4.6 In the figure, AC and BD intersect at point E. F is a point lying on BC. It is given that $\angle EBC = 40°$, $\angle AEF = 124°$ and $\angle DEF = 143°$. Find $\angle ECB$.

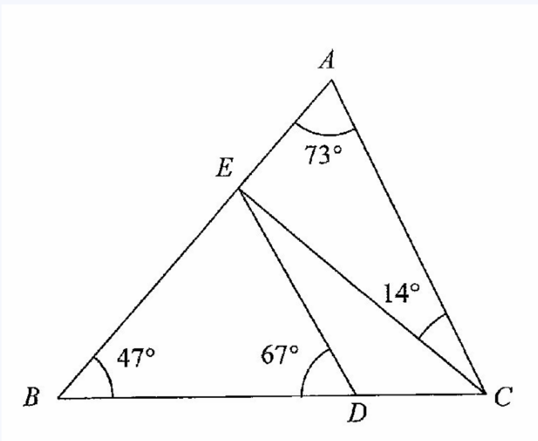


Exercise 4.7 Which of the following statements is correct?

A. If $\angle A + \angle B = 90°$, then they are supplementary angles.
B. An octagon has 10 sides.
C. All interior angles are obtuse in an obtuse-angled triangle.
D. A regular polygon must be equiangular.

Exercise 4.8 In the figure, ABC is a triangle and E and D are points on AB and BC respectively. It is given that $\angle ABC = 47°$, $\angle BAC = 73°$, $\angle ACE = 14°$ and $\angle BDE = 67°$.

(a) Find $\angle ECD$.
(b) Find $\angle DEC$.



4.4 DSE Corner

4.4.1 Long Questions

Problem 4.1 — 2020/I/Q8.

In Figure 1, B and D are points lying on AC and AE respectively. BE and CD intersect at the point F. It is given that $AB = BE$, $BD//CE$, $\angle CAE = 30°$ and $\angle ADB = 42°$.

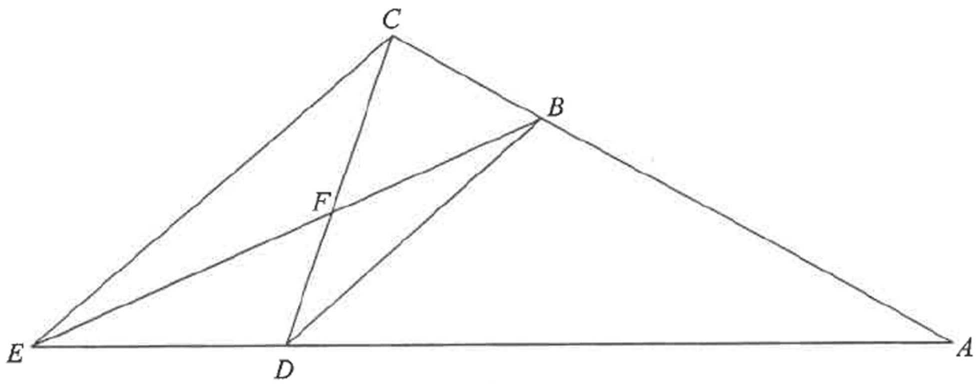


Figure 1

(a) Find $\angle BEC$.

(b) Let $\angle BDC = \theta$. Express $\angle CFE$ in terms of θ.

5. Congruence

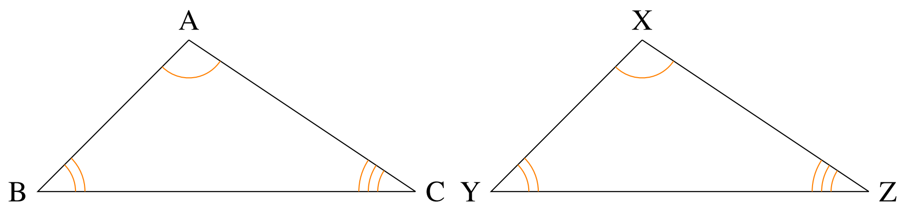
5.1 Congruent Triangles

Figures that have same shape and size are called congruent figures.

There are some properties of congruent triangles. If $\triangle ABC \cong \triangle XYZ$, then

(i) $\angle A = \angle X, \angle B = \angle Y, \angle C = \angle Z$ (corr. $\angle$s, $\cong \triangle$s)

(ii) $AB = XY, BC = YZ, CA = ZX$ (corr. sides, $\cong \triangle$s)



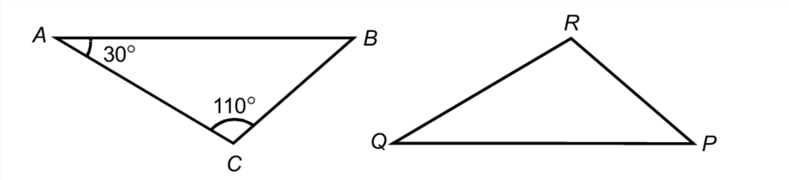
There are 5 conditions for congruent triangles:

1. Three Sides Equal (SSS)
2. Two Sides and Included Angle Equal (SAS)
3. Two Angles and Included Side Equal (ASA)
4. Two Angles and Non-included Side Equal (AAS)
5. Right Angle, Hypothenuse and One Side Equal (RHS)

SSS	SAS	ASA

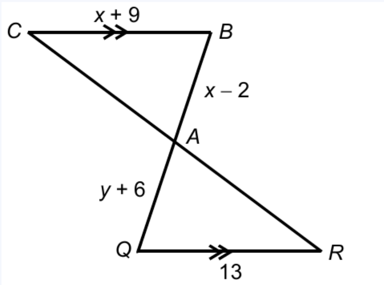
AAS	RHS

Exercise 5.1 If $\triangle ABC \cong \triangle PQR$, find $\angle Q$.



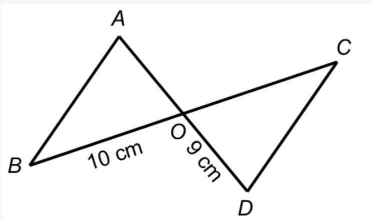
A. 30° B. 40° C. 100° D. 110°

Exercise 5.2 If $\triangle ABC \cong \triangle AQR$, find y.



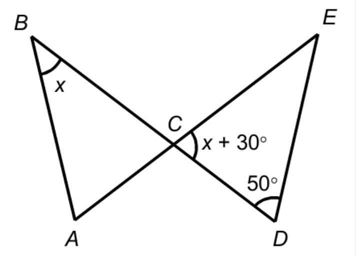
A. −4 B. −12 C. 4 D. 12

Exercise 5.3 If $\triangle ABO \cong \triangle DCO$, find the lenght of BC.



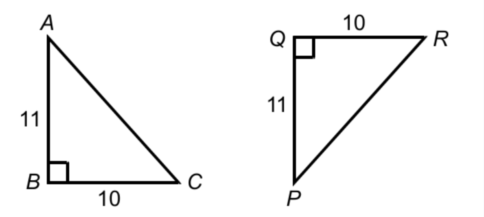
A. 18 cm B. 19 cm C. 20 cm D. 21 cm

Exercise 5.4 If $\triangle ABC \cong \triangle DEC$, find x.



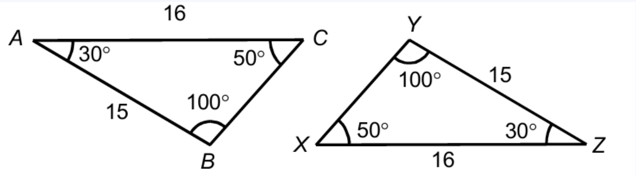
A. 20° B. 50° C. 60° D. 80°

Exercise 5.5 State the reason for the following triangles to be congruent.



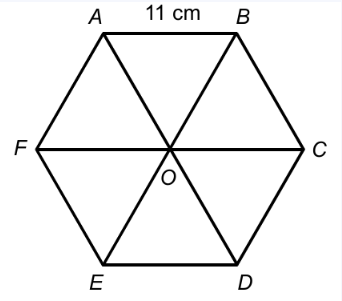
A. RHS B. SAS C. SSS D. ASA

Exercise 5.6 State the reason for the following triangles to be congruent.



A. RHS B. AAA C. SSS D. ASA

Exercise 5.7 The figure shows regular hexagon $ABCDEF$. Find the length of BO.



A. 10 cm B. 11 cm C. 12 cm D. Uncertain

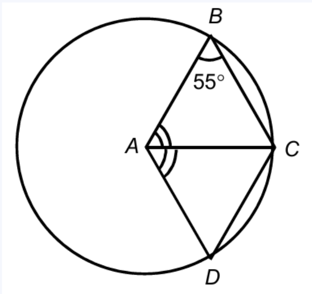
Exercise 5.8 Which of the following statements are correct?

I. Two squares with same area are congruent.

II. Two trapeziums with same area are congruent.
III. Two circles with same circumference are congruent.
IV. Two quadrilaterals with same perimeter are congruent.

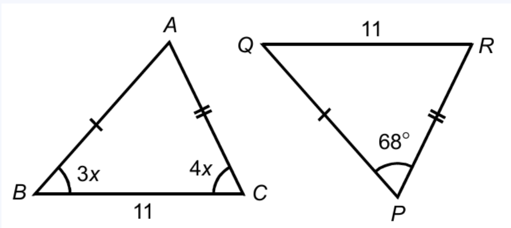
A. I and II
B. I and III
C. III and IV
D. I, II, III and IV

Exercise 5.9 The figure shows a circle with centre A. Find $\angle ADC$.



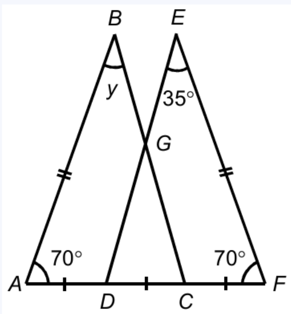
A. 35°　B. 55°　C. 70°　D. 110°

Exercise 5.10 In the figure, find x.



A. 9.7°　B. 16°　C. 17°　D. 22.7°

Exercise 5.11 In the figure, $ADCF$, BGC and EGD are straight lines. Find y.



A. 35°　B. 65°　C. 80°　D. 100°

6. More about Statistical Charts

6.1 Continuous Data in Statistics

We group a large number of continuous data into several *class intervals*. Then we have several terms:

1. *Class limit*: The end values of each class interval;
2. *Class mark*: The mid-value of each class interval;
3. *Class boundary*: The average of the upper class limit of one class interval and the lower class limit of the next class interval; and
4. *Class width*: The difference between the upper and lower class boundaries of each class interval.

For example, the following is the height of 20 people in a class.

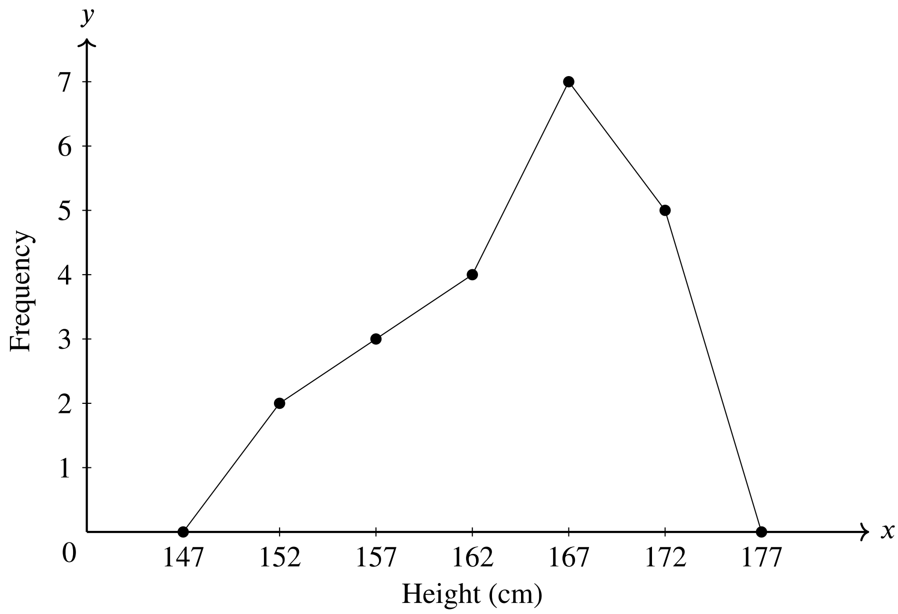
163	165	166	173	174	166	172	173	168	158
155	155	160	165	167	151	166	164	163	174

Height (cm)	**Class Boundaries (cm)**	**Class Marks (cm)**	**Frequency**
150 - 154	149.5 - 154.5	152	1
155 - 159	154.5 - 159.5	157	3
160 - 164	159.5 - 164.5	162	4
165 - 169	164.5 - 169.5	167	7
170 - 174	169.5 - 174.5	172	5

We can draw a *frequency polygon* by the use of above table. Also we have the *cummulative frequency polygon*. First we draft the cummulative frequency table as below.

Heights below (cm)	**154.5cm**	**159cm**	**164.5cm**	**169.5cm**	**174.5cm**
Frequency	1	4	8	15	20

Figure 6.1: Frequency Polygon



6.2 Usages of each Statistical Diagram

Different statistical diagrams provide different information. The following table summarises the characteristics of different statistical diagrams.

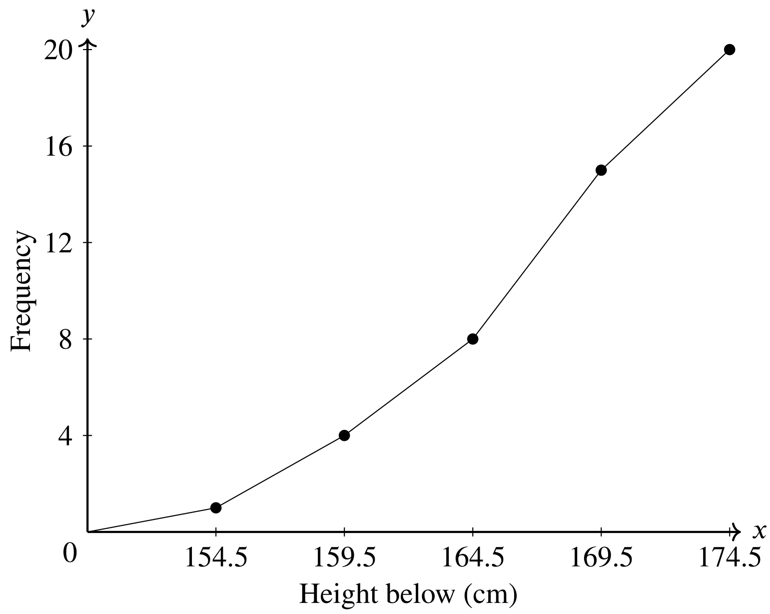
Statistical Diagram	Characteristics
Bar chart	show the actual frequency of each item
Pie chart	show the percentage of each item
Broken-line graph	show the change in frequencies of data over a period of time and the trend of data
Stem-and-leaf diagram	present a small amount of data exactly
Scatter diagram	show the relationship between two quantities
Histogram	show the frequency distribution of a set of continuous data
Frequency polygon or curve	show the frequency distribution of a set of continuous data
Cumulative frequency polygon of curve	show the cummulative frequency of the data up a certain value

6.3 Abuses of Statistical Diagrams

Some statistical diagrams may be misleading. We should pay attention to the following when reading statistical diagrams.

1. Whether the vertical axis starts from zero.
2. Whether the graphics used are of appropriate proportion.

Figure 6.2: Cummulative Frequency Polygon



Exercise 6.1 Which of the following is a continuous datum?

A. Length
B. Number of days
C. Quantity of paper
D. Level of building

Exercise 6.2 The following frequency distribution table shows the height of a group of students. How many students are 169.5 cm high or above?

Height (cm)	**Frequency**
140 – 149	7
150 – 159	2
160 – 169	5
170 – 179	8

A. 7
B. 8
C. 13
D. 14

Exercise 6.3 The following frequency distribution table shows the heights of 70 s tudents. How many students are 154.5 cm high or above?

Height (cm)	Frequency
135 – 139	16
140 – 144	10
145 – 149	15
150 – 154	6
155 – 159	18
160 – 164	5

A. 6 B. 23 C. 29 D. 47

Exercise 6.4 The following frequency distribution table shows the monthly salaries of the workers in a factory. Find the number of workers whose monthly salaries are less than \$6 750.

Monthly salary (\$)	Frequency
3300 – 3700	1
3800 – 4200	3
4300 – 4700	5
4800 – 5200	10
5300 – 5700	13
5800 – 6200	8
6300 – 6700	10

A. 10 B. 30 C. 40 D. 50

Exercise 6.5 The following frequency distribution table shows the Mathematics examination scores of 50 students. What percentage of students got 59.5 or higher?

Score	Frequency
0 – 19	2
20 – 39	7
30 – 59	11
60 – 79	14
80 – 99	16
100	0
Total	50

A. 4% B. 18% C. 40% D. 60%

Exercise 6.6 The following frequency distribution table shows the weights of a group of students. Find the class mark of the third class.

Weight (kg)	Frequency
35 – 39	4
40 – 44	7
45 – 49	9
50 – 54	13

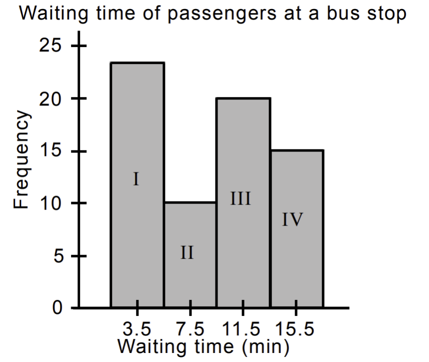
A. 46 kg B. 46.5 kg C. 47 kg D. 47.5 kg

Exercise 6.7 The following frequency distribution table shows the daily exercise time of a class of students. Find the lower class boundary and upper class boundary of the 8th class.

Time (min)	Frequency
1 – 15	5
16 – 30	7
31 – 45	3
46 – 60	4
61 – 75	2
76 – 90	9
91 – 105	2
106 – 120	10

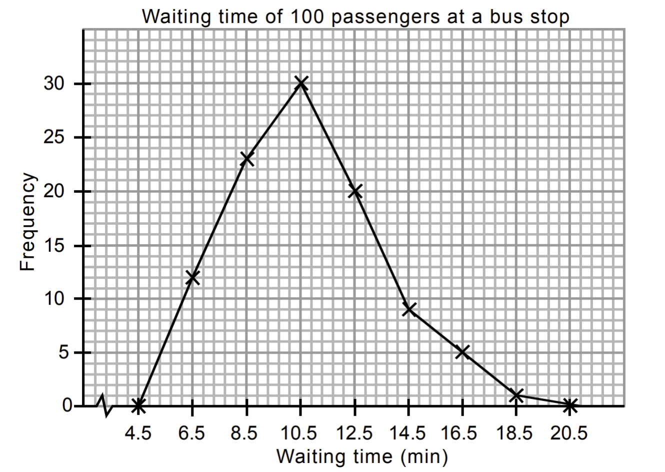
A. Lower class boundary = 105.5 min; upper class boundary = 120.5 min
B. Lower class boundary = 106 min; upper class boundary = 120 min
C. Lower class boundary = 91 min; upper class boundary = 105 min
D. Lower class boundary = 90.5 min; upper class boundary = 105.5 min

Exercise 6.8 The histogram shows the waiting time of passengers at a bus stop. Which class has the most passengers?



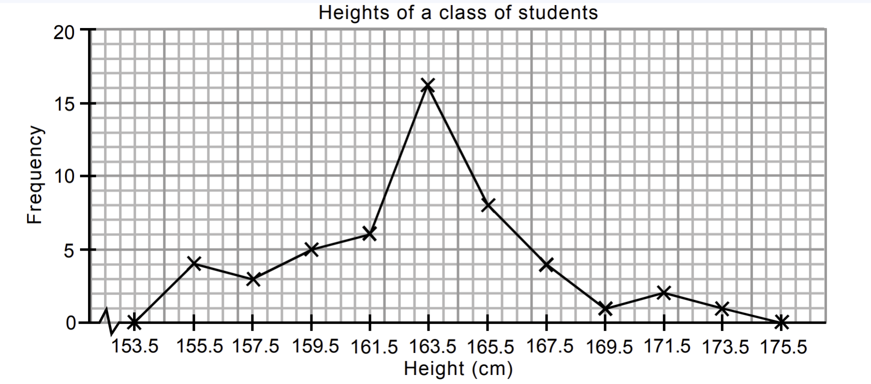
A. I B. II C. III D. IV

Exercise 6.9 The following frequency polygon shows the waiting time of 100 passengers at a bus stop. How many passengers are there in the class interval 18 min – 19 min?



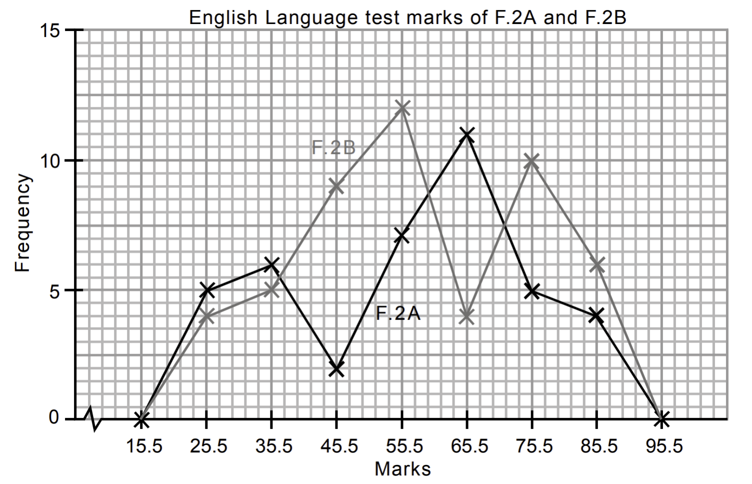
A. 0 B. 1 C. 2 D. 5

Exercise 6.10 The following frequency polygon shows the heights of a class of students. What percentage of students is shorter than 168.5 cm?



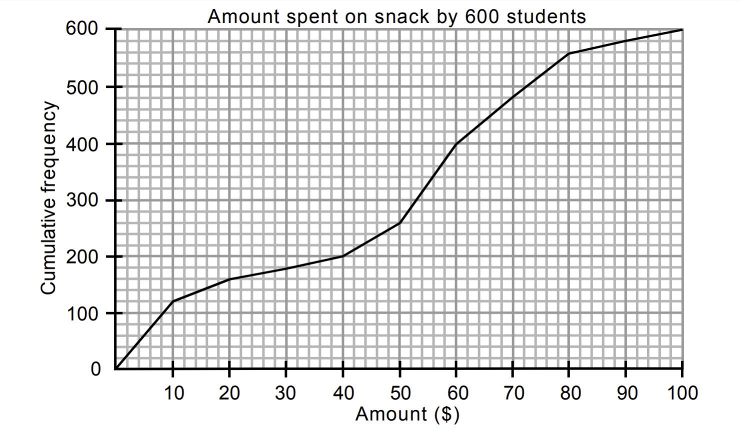
A. 8% B. 10% C. 84% D. 92%

Exercise 6.11 The following frequency polygons show the English Language test marks of F.2A and F.2B students. If the passing mark is 60, which class has a higher passing percentage in the English Language test?



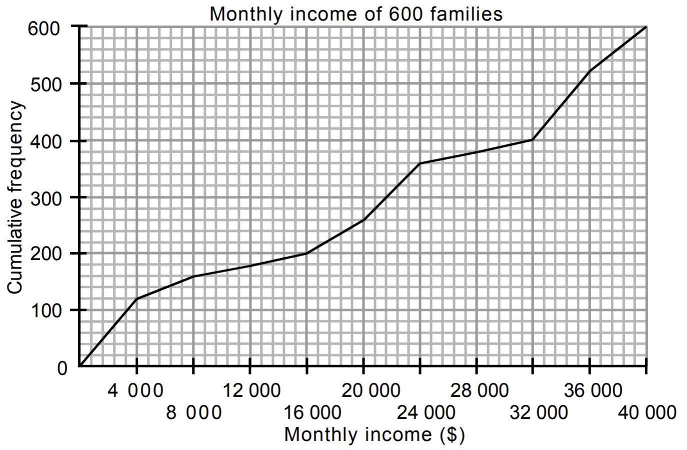
A. Both classes have the same passing percentage in the English Language test.
B. It cannot be found.
C. F.2A
D. F.2B

Exercise 6.12 The following cumulative frequency polygon shows the amounts spent on snack by 600 students. Find the difference between the upper quartile and the lower quartile.



A. $25 B. $26 C. $49 D. $50

Exercise 6.13 The following cumulative frequency polygon shows the monthly income of 600 families in a certain public housing estate. If the top 10% of the families have to pay double housing rent, what is the minimum monthly income of these families?



A. $2 000 B. $36 000 C. $36 800 D. $38 000

Exercise 6.14 The table below shows the distribution of the amount of pocket money of F.2B students per week.

Pocket money ($)	Number of students
50 – 149	5
150 – 249	21
250 – 349	10
350 – 449	4

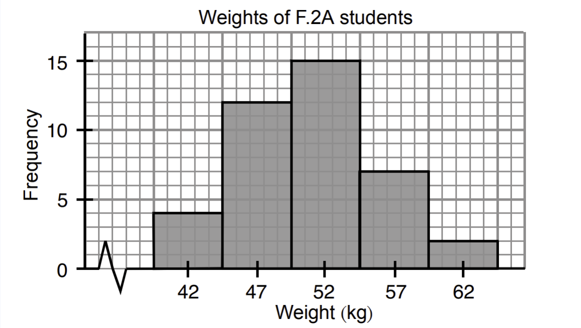
(a) How many students have pocket money less than $149.5 per week?
(b) Find the lower class limit and the upper class limit of the second class.
(c) Find the lower class boundary and the upper class boundary of the third class.
(d) Find the class mark of the fourth class.

Exercise 6.15 The table below shows the Mathematics quiz marks of F.2C students. (Full marks is 20.)

Pocket money ($)	Number of students
1 – 5	3
6 – 10	7
11 – 15	18
16 – 20	12

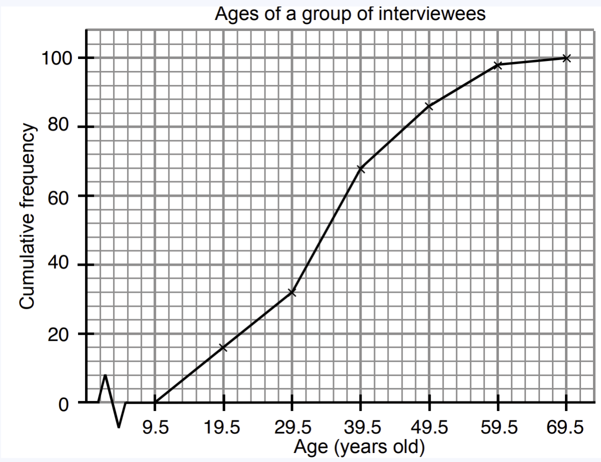
(a) Find the class interval of the second class.
(b) Find the upper class limit of the first class.
(c) How many students have less than 5.5 marks?
(d) Find the lower class limit of the third class.
(e) How many students are there in F.2C?

Exercise 6.16 The following histogram shows the weights of F.2A students.



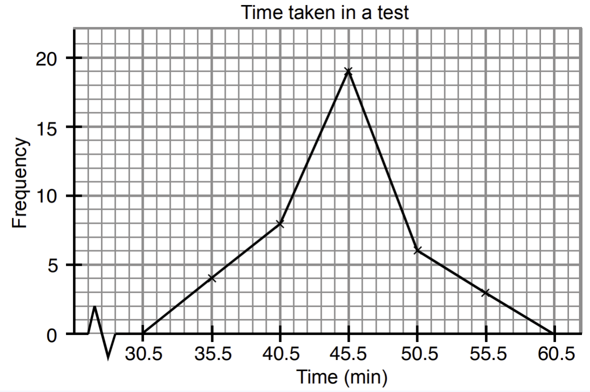
(a) Which class interval has the highest frequency?
(b) Find the number of students in the class interval 55 kg – 59 kg.
(c) How many students are there in the class interval with the lowest frequency?

Exercise 6.17 The following cumulative frequency curve shows the ages of a group of interviewees.



(a) Find the number of interviewees whose ages are between 29.5 and 49.5.
(b) What percentage of interviewees are under 39.5 years old?

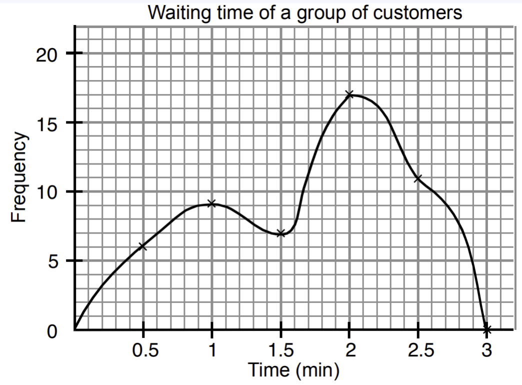
Exercise 6.18 The following frequency polygon shows the time taken in a 60-minute test by a group students.



(a) How many students had finished the test 9.5 minutes before the test ended?

(b) If bonus marks will be awarded to students who finish the test within 40.5 minutes, what percentage of students can get the bonus marks?

Exercise 6.19 The following frequency curve shows the waiting time of a group of customers of Great Happy Fast Food Shop.

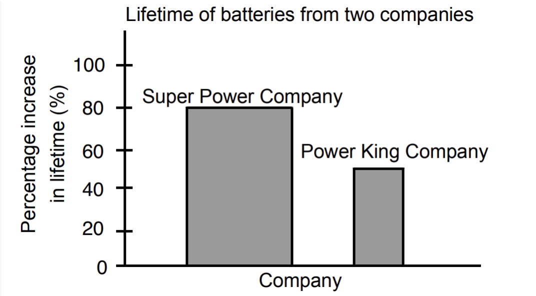


(a) The owner of the fast food shop claims that the average waiting time of customers is 1.5 minutes. Is that true?

(b) Nahida waited for 2 minutes and 45 seconds. Did she wait too long in comparison with most of the customers?

(c) What percentage of customers waited less than 45 seconds?

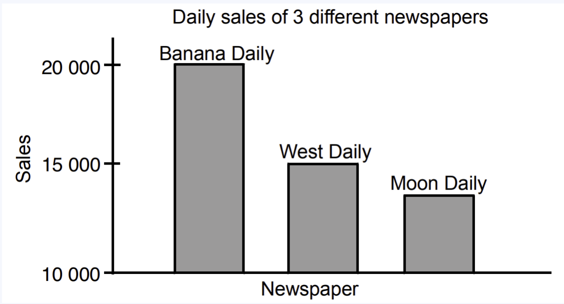
Exercise 6.20 Super Power Company and Power King Company manufacture their respective new batteries. The spokesman of Super Power Company claims that the lifetime of its new battery has increased by 80%, but the lifetime of the new battery of Power King Company has just increased by 50%. Hence, he concludes that the new battery manufactured by his company is more long-lasting. He also explains his claim with the following graph.



(a) What is the mistake of the bar chart?

(b) What is the mistake of his claim?

Exercise 6.21 The following bar chart shows the average daily sales of 3 different newspapers.



(a) Which newspaper has the best sales?

(b) Dick claims that the sales of Banana Daily is twice the sales of West Daily.

   (i) Is he correct?

   (ii) Let the sales of Banana Daily be x and that of West Daily be y. Find x : y. Hence, amend Dick's claim.

(c) Does the graph mislead readers? Explain.

Book 2B

7. Rate, Ratio and Proportion

7.1 Rates

A *rate* is a comparison of two quantities of *different* kinds by division. Rates have *units*. It represents the relationship of the amount of one quantity per unit of another quantity. For example, a car is driving 80 km within 2 hours, then its speed, a kind of rate, is 40 km/h.

Exercise 7.1 A car travels 162 km in 2 hours. Find the speed of the car in each of the following units.

(a) km / h

(b) m / s

Exercise 7.2 A printer can print 135 A4 size pages in 15 minutes.

(a) Find the printing rate of the printer in pages / min.

(b) How many A4 size pages can be printed in 40 seconds?

Exercise 7.3 Currently, the exchange rate between Hong Kong Dollar (HKD) and Japanese Yen (JPY) is 19.43 JPY / HKD.

(a) Find the amount of JPY that can be exchanged for HKD 40?

(b) Find the amount of HKD that can be exchanged for JPY 100?

7.2 Ratios

A *ratio* is a comparison of quantities of *the same* kind by division. Ratios have *no units*. For two quantities a and b, the ratio of a to b can be denoted as $a : b$ or $\frac{a}{b}$, where $a \neq 0$ and $b \neq 0$. For example, 10 mL household bleach should be mixed with 990 mL water before use, then the ratio of bleach to water is $1 : 99$. Note that $a : b = 2 : 3$ does not necessarily mean that $a = 2$ and $b = 3$.

The ratio of three or more quantities is called a *continued ratio*. If two related ratios, e.g. $a : b$ and $b : c$ are given, we can combine the two ratios into a continued ratio $a : b : c$. For example, if $a : b = 3 : 4$ and $b : c = 4 : 9$, then $a : b : c = 3 : 4 : 9$. Note that it cannot be expressed as a fraction.

When each quantity of a ratio is multiplied or divided by the same non-zero number k, the ratio remains unchanged.

Exercise 7.4 Determine the following statements is true or false.
(a) If $\frac{a}{b} = 1.25$, then $a : b = 1.25 : 1$.
(b) If $x : y : z = 1 : 2 : 6$, then $x = 1$, $y = 2$ and $z = 6$.

Exercise 7.5 Simplify the following ratios.
(a) 64 : 24
(b) 0.35 : 2.1
(c) $0.4 : \frac{6}{4} : 1$
(d) 1.2 km : 240 cm
(e) 0.35 kg : 6300 g
(f) 3.2 hours : 1.5 days

Exercise 7.6 For three kinds of tickets in a flight: first class, business class and economy class, which costs \$10 000, \$6 000 and \$3 000 respectively. The ratio between the number of tickets sold for first class and that for business class is $1 : 3$, while the ratio between the number of tickets sold for business class and that for economy class is $1 : 13$. If the total revenue from selling first class and economy class is \$254 000.
(a) Find the total number of tickets sold for the flight.
(b) Find the total revenue from selling business class.

Exercise 7.7 Find $x : y$ in each of the following.
(a) $6y - 11x = 0$
(b) $4x - 3y = x + 10y$
(c) $(x + 2y) : (4y - x) = 3 : 4$

7.3 Proportions

A *proportion* is an equation in which two ratios are set equal to each other. A scale drawing is a reduced or an enlarged drawing of a real object according to a specific ratio called scale.

$$\text{Scale} = \frac{\text{length on the scale drawing}}{\text{actual length}}$$

For proportion, there is *direct proportion* and *indirect proportion*. Two quantities are in direct proportion when they increase or decrease at the same rate, while they are in inverse proportion when one quantity increases as the other decreases and vice versa, in such way, their product is a constant. Mathmeatically, for some constant k,

$$\text{Direct Proportion: } y = kx$$

$$\text{Inverse Proportion: } xy = k, \text{or} y = \frac{k}{x}$$

Exercise 7.8 Assume a plant will produce certain amount of oxygen, which is directly proportional to the amount of glucose produced and inversely proportional to the amount of carbon dioxide absorbed. If the amount of glucose produced is 4 (mol) and the amount of carbon dioxide absorbed is 200 (mol), then 20 (mol) of oxygen is produced.

(a) If the amount of glucose produced is 3 (mol), what is the amount of oxygen produced?
(b) If the amount of carbon dioxide absorbed is 375 (mol), what is the amount of oxygen produced?
(c) If the amount of oxygen produced is 16 (mol), what is the amount of glucose produced and the amount of carbon dioxide produced?

7.4 DSE Corner

7.4.1 Long Questions

Problem 7.1 — PP/I/Q5.

The ratio of the capacity of a bottle to that of a cup is 4 : 3. The total capacity of 7 bottles and 9 cups is 11 litres. Find the capacity of a bottle.

Problem 7.2 — 2018/I/Q9.
A car travels from city P to city Q at an average speed of 72 km / h and then the car travels from city Q to city R at an average speed of 90 km / h. It is given that the car travels 210 km in 161 minutes for the whole journey. How long does the car take to travel from city P to city Q?

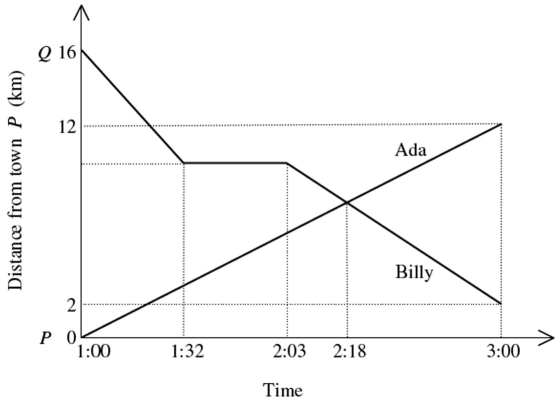
Problem 7.3 — 2019/I/Q7.
In a playground, the ratio of the number of adults to the number of children is 13 : 6. If 9 adults and 24 children enter the playground, then the ratio of the number of adults to the number of children is 8 : 7. Find the original number of adults in the playground.

Problem 7.4 — 2020/I/Q4.
Let a, b and c be non-zero numbers such that $\frac{a}{b} = \frac{6}{7}$ and $3a = 4c$. Find $\frac{b+2c}{a+2b}$.

Problem 7.5 — 2023/I/Q6.
Let a, b and c be non-zero numbers such that $7a = 6b$ and $\frac{4a-3c}{2b-c} = 9$. Find

(a) $a : b : c$,
(b) $\frac{5a+8b}{7b+3c}$.

Problem 7.6 — PP/I/Q12.
The figure shows the graphs for Ada and Billy running on the same straight road between town P and town Q during the period 1:00 to 3:00 in an afternoon. Ada runs at a constant speed. It is given that town P and town Q are 16 km apart.



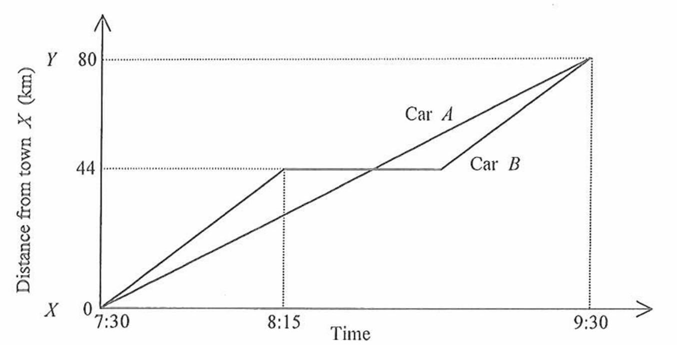
(a) How long does Billy rest during the period?

(b) How far from town P do Ada and Billy meet during the period?

(c) Use average speed during the period to determine who runs faster. Explain your answer.

Problem 7.7 — 2014/I/Q10.

Town X and town Y are 80 km apart. The figure shows the graphs for car A and car B travelling on the same straight road between town X and town Y during the period 7:30 to 9:30 in a morning. Car A travels at a constant speed during the period. Car B comes to rest at 8:15 in the morning.



(a) Find the distance of car A from town X at 8:15 in the morning.

(b) At what time after 7:30 in the morning do car A and car B first meet?

(c) The driver of car B claims that the average speed of car B is higher than that of car A during the period 8:15 to 9:30 in the mroning. Do you agree? Explain your answer.

7.4.2 Multiple Choice Questions

Problem 7.8 — 2012/II/Q11.

Mary performs a typing task for 7 hours. Her average typing speeds for the first 3 hours and the last 4 hours are 63 words per minute and 56 words per minute respectively. Find her average typing speed for the 7 hours.

A. 17 words per minute

B. 35 words per minute

C. 59 words per minute

D. 60 words per minute

Problem 7.9 — SP/II/Q13.

If x, y and z are non-zero numbers such that $2x = 3y$ and $x = 2z$, then $(x+z) : (x+y) =$

A. 3 : 5. B. 6 : 7. C. 9 : 7. D. 9 : 10.

Problem 7.10 — PP/II/Q12.
If α and β are non-zero numbers such that $(\alpha+\beta):(3\alpha-\beta)=7:3$, then $\alpha:\beta=$

A. 5 : 9. B. 9 : 5. C. 19 : 29. D. 29 : 19.

Problem 7.11 — 2012/II/Q9.
If x and y are non-zero numbers such that $\dfrac{6x+5y}{3y-2x}=7$, then $x:y=$

A. 4 : 5. B. 4 : 13. C. 5 : 4. D. 13 : 4.

Problem 7.12 — 2013/II/Q12.
The actual area of a playground is 900 m^2. If the area of the playground on a map is 36 cm^2, then the scale of the map is

A. 1 : 25. B. 1 : 50. C. 1 : 500. D. 1 : 250 000.

Problem 7.13 — 2014/II/Q12.
It is given that $\dfrac{4}{5a}=\dfrac{5}{7b}=\dfrac{7}{9c}$, where a, b and c are positive numbers. Which of the following is true?

A. $a<b<c$ B. $a<c<b$ C. $b<a<c$ D. $b<c<a$

Problem 7.14 — 2015/II/Q11.
If a, b and c are non-zero numbers such that $a:c=5:3$ and $b:c=3:2$, then $(a+c):(b+c)=$

A. 7 : 5. B. 8 : 5. C. 16 : 15. D. 19 : 15.

Problem 7.15 — 2016/II/Q11.
If x and y are non-zero numbers such that $(3y-4x):(2x+y)=5:6$, then $x:y=$

A. 7 : 8. B. 8 : 29. C. 9 : 32. D. 13 : 34.

Problem 7.16 — 2016/II/Q13.
The cost of flour of brand X is \$42 / kg. If 3 kg of flour of brand X and 2 kg of flour of brand Y are mixed so that the cost of the mixture is \$36 / kg, find the cost of the flour of brand Y.

A. \$27 / kg B. \$30 / kg C. \$32 / kg D. \$39 / kg

Problem 7.17 — 2017/II/Q11.
The scale of a map is 1 : 20 000. If the area of a zoo on the map is 4 cm^2, then the actual area of the zoo is

A. 8×10^4 m^2. B. 1.6×10^5 m^2. C. 3.2×10^5 m^2. D. 1×10^6 m^2.

Problem 7.18 — 2018/II/Q10.
If a, b and c are non-zero numbers such that $3a=4b$ and $a:c=2:5$, then $\dfrac{a+3b}{b+3c}=$

A. $\dfrac{5}{3}$. B. $\dfrac{13}{33}$. C. $\dfrac{30}{53}$. D. $\dfrac{75}{38}$.

Problem 7.19 — 2019/II/Q12.
The cost of tea of brand A and B are \$140 / kg and \$315 / kg respectively. If x kg of tea of brand A

and y kg of tea of brand B are mixed so that the cost of the mixture is \$210 / kg, then $x : y =$.

A. 2 : 3. B. 3 : 2. C. 4 : 9. D. 9 : 4.

Problem 7.20 — 2020/II/Q10.

The actual area of a golf courses is 0.75 km^2. If the area of the course on a map is 300 cm^2, then the scale of the map is

A. 1 : 250. C. 1 : 62 500. 000.
B. 1 : 5 000. D. 1 : 25 000

Problem 7.21 — 2021/II/Q11.

If α and β are non-zero numbers such that $\frac{2\alpha+3\beta}{3\alpha+2\beta} = \frac{7}{10}$, then $\frac{2\alpha+\beta}{\alpha+2\beta} =$

A. 1. B. $\frac{3}{2}$. C. $\frac{11}{6}$. D. $\frac{13}{8}$.

Problem 7.22 — 2022/II/Q12.

Let x, y and z be non-zero numbers. If $x : y = 8 : 5$ and $2x = 4z - 3y$, then $y : z =$

A. 16 : 17. B. 17 : 16. C. 20 : 31. D. 31 : 20.

Problem 7.23 — 2023/II/Q12.

The scale of a map is 1 : 50 000. If the actual area of an airport is 10 km^2, then the area of this airport on the map is

A. 2 cm^2. B. 4 cm^2. C. 20 cm^2. D. 40 cm^2.

8. Similarity

8.1 Similar Triangles

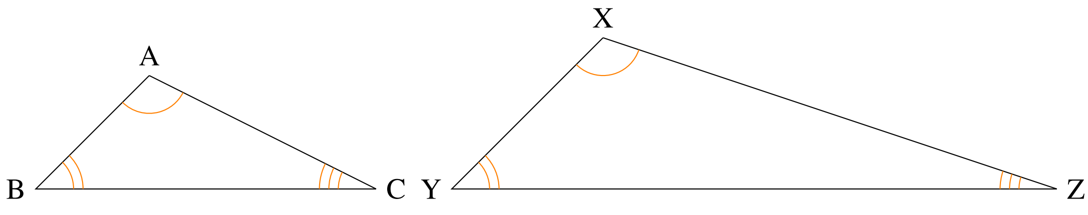
Figures that have the same shape but not necessarily the same size are called similar figures.

R Congruent figures are also similar figures.

There is some properties of similar triangles: If $\triangle ABC \sim \triangle XYZ$, then

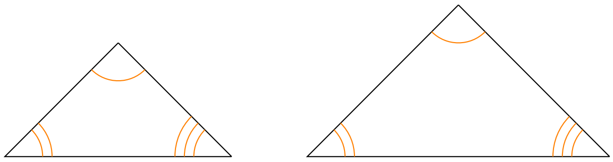
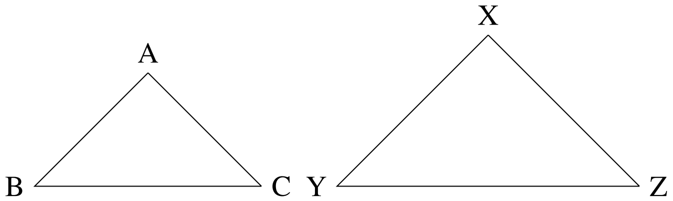
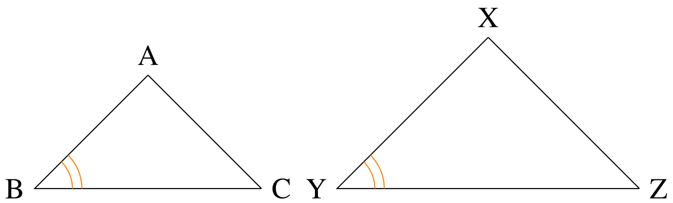
(i) $\angle A = \angle X, \angle B = \angle Y, \angle C = \angle Z$ (corr. $\angle$s, $\sim \triangle$s)

(ii) $\frac{XY}{AB} = \frac{YZ}{BC} = \frac{ZX}{CA}$ (corr. sides, $\sim \triangle$s)

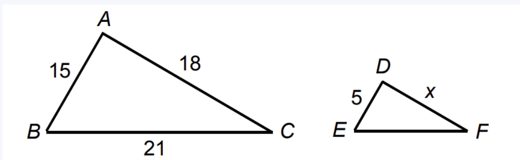


Similar to congruent triangles, there are 3 conditions for similar triangles.

1. Three Angles Equal (AAA)
2. Three Sides Proportional (3 sides prop.)
3. Ratio of Two Sides and an Included Angle Equal (ratio of 2 sides, inc. $\angle$)

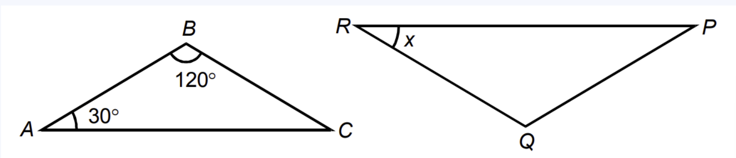
Three Angles Equal (AAA)

Three Sides Proportional (3 sides prop.)
 $\frac{XY}{AB} = \frac{YZ}{BC} = \frac{ZX}{CA}$
Ratio of Two Sides and an Included Angle Equal (ratio of 2 sides, inc. ∠)
 $\frac{XY}{AB} = \frac{YZ}{BC}$ and $\angle B = \angle Y$

Exercise 8.1 If $\triangle ABC \sim \triangle DEF$, find x.



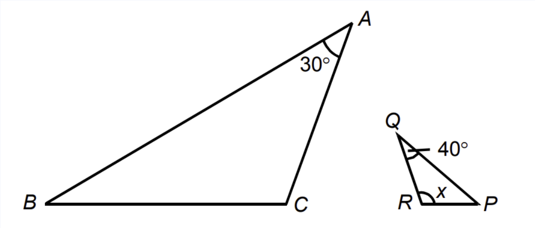
A. 6 B. 7 C. 10 D. 13

Exercise 8.2 If $\triangle ABC \sim \triangle PQR$, find x.



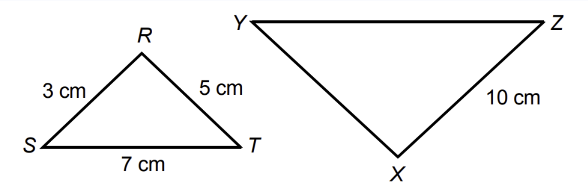
A. 10° B. 30° C. 50° D. 80°

Exercise 8.3 If $\triangle ABC \sim \triangle PQR$, find x.



A. 30° B. 40° C. 110° D. Uncertain

Exercise 8.4 If $\triangle RST \sim \triangle XYZ$, find the length of YZ.

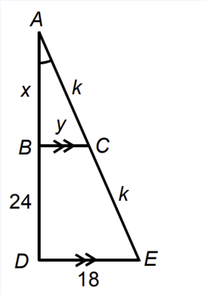


A. 10 cm B. 12 cm C. 14 cm D. 15 cm

Exercise 8.5 Given $\triangle XYZ \sim \triangle PQR$. If $\angle X = 60°$ and $\angle Z = 32°$, find $\angle Q$.

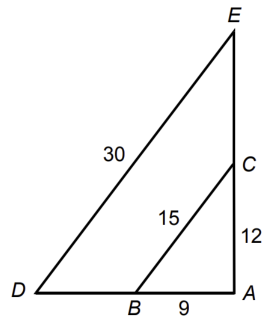
A. 78° B. 88° C. 98° D. 108°

Exercise 8.6 If $\triangle ABC \sim \triangle ADE$, find y.



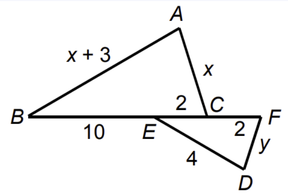
A. 9 B. 14 C. $x+2k$ D. $\dfrac{18x}{x+24}$

Exercise 8.7 If $\triangle ABC \sim \triangle ADE$, find the length of CE.



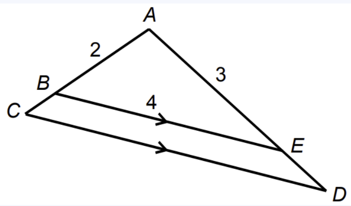
A. 6 B. 9 C. 12 D. 18

Exercise 8.8 If $\triangle ABC \sim \triangle DEF$, find y.



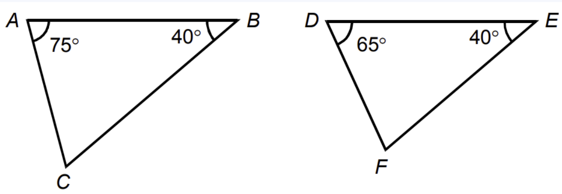
A. 3 B. 9 C. $\frac{1}{9}$ D. $\frac{1}{3}$

Exercise 8.9 Is the following pair of triangles similar? If yes, state the reason.



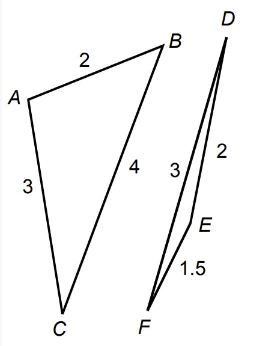
A. Yes, equiangular.
B. Yes, 3 sides proportional.
C. Yes, ratio of 2 sides, inc. ∠.
D. No.

Exercise 8.10 Is the following pair of triangles similar? If yes, state the reason.



A. Yes, equiangular.
B. Yes, 3 sides proportional.
C. Yes, ratio of 2 sides, inc. ∠.
D. No.

Exercise 8.11 Is the following pair of triangles similar? If yes, state the reason.



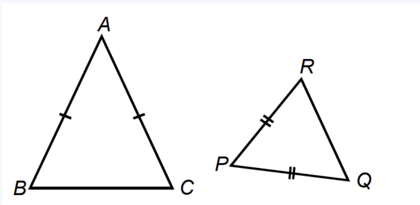
A. Yes, equiangular.
B. Yes, 3 sides proportional.
C. Yes, ratio of 2 sides, inc. ∠.
D. No.

Exercise 8.12 Which of the following statements are correct?

I Squares with different perimeters are similar.
II Rhombuses with different areas are similar.
III Right-angled triangles with different perimeters are similar.
IV Circles with different areas are similar.

A. I and III
B. I and IV
C. II and III
D. III and IV

Exercise 8.13 What additional information is required so that $\triangle ABC \sim \triangle PQR$?

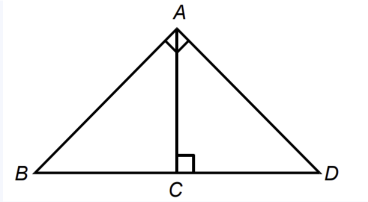


A. $\angle A = \angle R$
B. $AC = PR$
C. $BC = QR$
D. $\angle A = \angle P$

Exercise 8.14 If $\triangle ABC \sim \triangle PQR$, which of the following must be true?

A. $\angle ACB = \angle QRP, \dfrac{BC}{QR} = \dfrac{AC}{PR}$
B. $\angle BCA = \angle QRP, \dfrac{AB}{PQ} = \dfrac{PR}{AC}$
C. $\angle A = \angle P, \angle B = \angle R, \angle C = \angle P$
D. $AB = PQ, BC = QR, AC = PR$

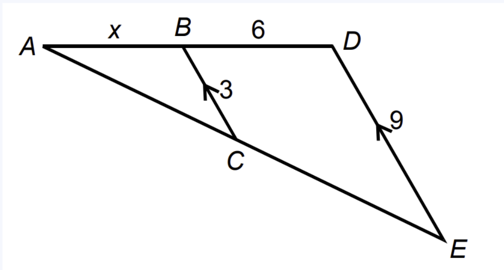
Exercise 8.15 In the figure, BCD is a straight line. Which of the following are true?



I. $\triangle ADB \sim \triangle CAB$ II. $\triangle ACD \sim \triangle ABD$ III. $\triangle ABC \sim \triangle DBA$

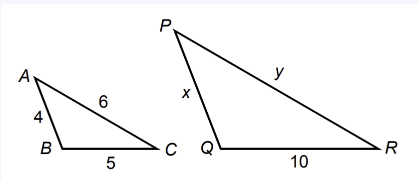
A. I and II B. I and III C. II and III D. I, II and III

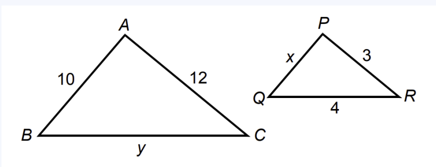
Exercise 8.16 In the figure, ABD and ACE are straight lines. Which of the following formulae can be used to find x?

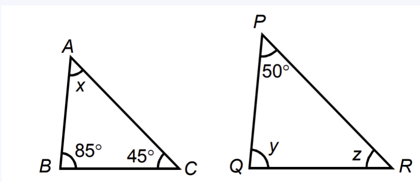


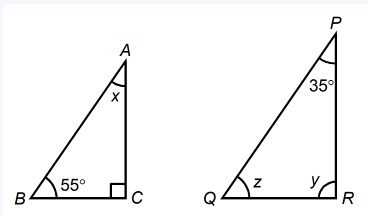
A. $\dfrac{x}{x+6} = \dfrac{3}{9}$ B. $\dfrac{9}{3} = \dfrac{x}{x+6}$ C. $\dfrac{x}{6} = \dfrac{3}{9}$ D. $\dfrac{9}{3} = \dfrac{x}{6}$

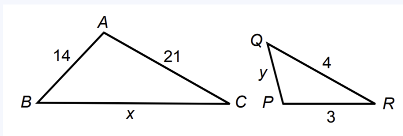
Exercise 8.17 In each of the following, $\triangle ABC \sim \triangle PQR$. Find the unknowns. (The figures are not drawn in scale.)

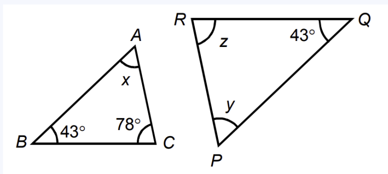
(a)


(b)


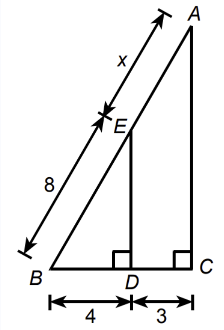
(c)


(d)


(e)


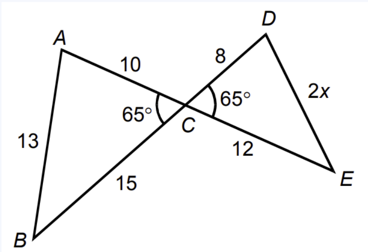
(f)


Exercise 8.18 In the figure, ABC is a straight line.



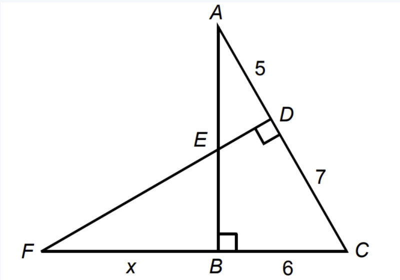
(a) Write down a pair of similar triangles and state the reason.
(b) Find x.

Exercise 8.19 In the figure, ABC is a straight line.



(a) Write down a pair of similar triangles and state the reason.
(b) Find x.

Exercise 8.20 In the figure, ADC, AEB, FBC and FED are straight lines.



(a) Write down a pair of similar triangles and state the reason.
(b) Find x.

9. Linear Equations in Two Unknowns

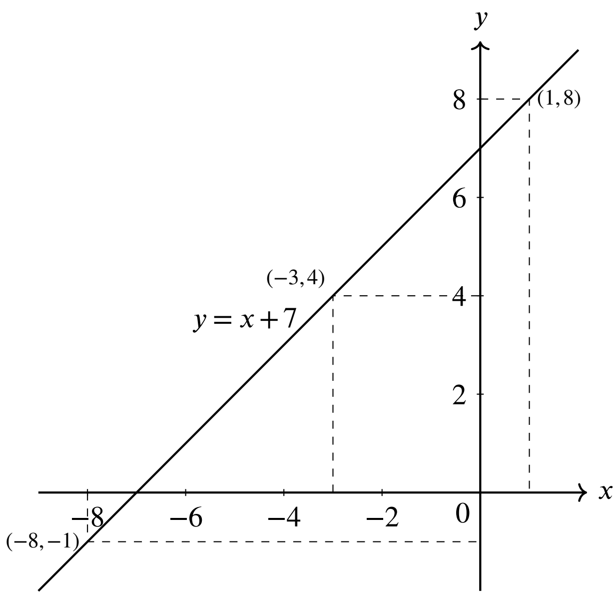
9.1 Linear Equations in Unknowns

9.1.1 Linear Equations

To learn linear equations in two unknowns, we first have to know what are linear equations. *Linear equations* are the equations in which the highest power of the variable is 1. For example, $3x+5=8$ and $x=2$. Similarly, there is something called *quardratic equations* in which the highest power of the variable is 2 and *cubic equations* in which the highest power of the variable is 3.

9.1.2 Linear Equations in Two Unknowns

Then we know *linear equations in two unknowns* means that there are two variables and each variable's power is 1. For example, $y=3x+8$ and $x+y=4$. A linear equations in two unknowns can be plotted on a graph. If there is only one linear equations in two unknowns, there is infinitely many solutions. For example, in $y=x+7$, $(1,8)$ and $(-8,-1)$ are possible solutions.



9.2 Simultaneous Linear Equations in Two Unknowns

If two equations in two unknowns are required to satisfied simultaneously, there is at most one pair of solutions, and this is called the simultaneous equations. For example, $(3,2)$ is the solutions of $3x+y=11$ and $2x+y=8$. Note that it can be no solutions. For example, $x+y=4$ and $x+y=5$ have no solutions. Geographically, there is no intersections between them, or their slope are the same.

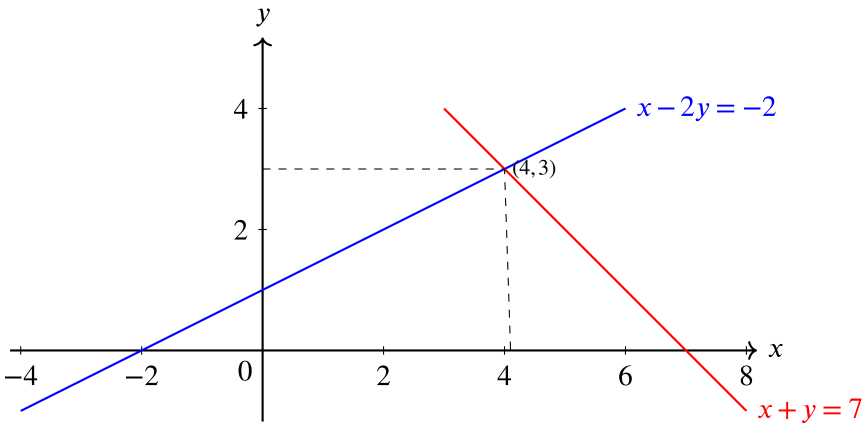
9.2.1 Methods to solve Simultaneous Equations

To solve the simultaneous equations, other than graphical method, we have two other methods, namely elimination and substitution. Substitution method is almost the most powerful method but sometimes clumsy, while elimination sometimes is better.

■ **Example 9.1**

Consider $\begin{cases} x+y &= 7 \\ x-2y &= -2 \end{cases}$.

1. Graphical method:



2. Method of elimination:

$$\begin{cases} x+y &= 7 \\ x-2y &= -2 \end{cases} = \begin{cases} 3x &= 12 \\ x-2y &= -2 \end{cases} = \begin{cases} x &= 4 \\ y &= 3 \end{cases}$$

3. Method of substitution:

$$\begin{cases} x+y &= 7 \\ x-2y &= -2 \end{cases} = \begin{cases} x &= 7-y \\ (7-y)-2y &= -2 \end{cases} = \begin{cases} x=7-3 &= 4 \\ y &= 3 \end{cases}$$

■

Exercise 9.1 Which of the following sets of values satisfies the equation $x-y=9$?

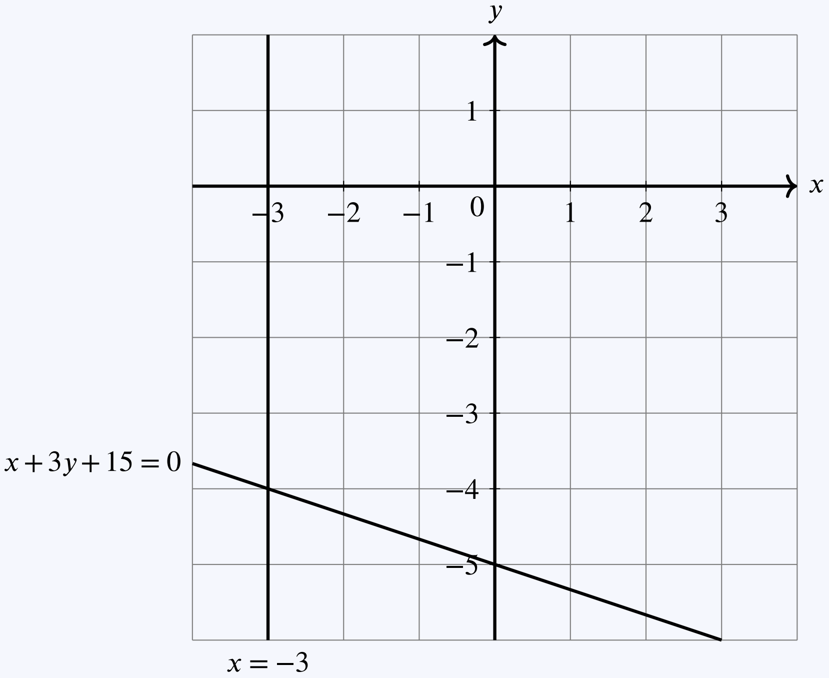
A. $x=1, y=9$

B. $x=9, y=-1$

C. $(x,y)=(8,1)$

D. $(x,y)=(8,-1)$

Exercise 9.2 Solve the simultaneous equations $\begin{cases} x+3y+15 &= 0 \\ x &= -3 \end{cases}$ graphically.



A. $x=-3, y=-4$

B. $x=-4, y=-3$

C. $x=3, y=-4$

D. $x=-3, y=4$

Exercise 9.3 Which of the following equations is impossible to be derived when solving $\begin{cases} 2x+y &= 8 \\ x-2y &= 9 \end{cases}$ by substitution.

A. $x=9+2y$

B. $x=\dfrac{8-y}{2}$

C. $y=-8+2x$

D. $y=\dfrac{x-9}{2}$

Exercise 9.4 Solve $2x-4y+11=5x-3y-5=3$.

A. $(5,4.5)$ B. $(0,2)$ C. $(1.6,0)$ D. $(4,4)$

Exercise 9.5 Which of the following simultaneous equations have no solutions?

A. $2x-3=y=5x-3$

B. $y=4x-5=2x-3$

C. $3x-1=4x-1=2y$

D. $x-2=x-7=y$

Exercise 9.6 How many solutions do the simultaneous equations $\begin{cases} 2x+2y &= 8 \\ x+y &= 4 \end{cases}$ have?

A. ∞ B. 2 C. 1 D. 0

Exercise 9.7 If $3y = x$ and $4x - 3y = 9$, find the value of $x + y$.

A. 1 B. 2 C. 3 D. 4

Exercise 9.8 If $2x = y$ and $x + 4y = 3$, find the value of xy.

A. $\frac{1}{9}$ B. $\frac{2}{9}$ C. $\frac{1}{3}$ D. $\frac{4}{9}$

Exercise 9.9 If $x : y = 2 : 1$ and $x + 4y = 3$, find the values of x and y.

A. $(0.5, 1)$ B. $(1, 0.5)$ C. $(2, 1)$ D. $(4, 2)$

Exercise 9.10 If the sum of two numbers is 54 and their difference is 20, find the two numbers

A. (17, 37) B. (14, 40) C. (24, 30) D. (20, 34)

Exercise 9.11 If the $(x, y) : (4, 0)$ and $(-2, 1)$ satisfy the equation $mx - ny = 4$, find (m, n).

A. $(1, -6)$ B. $(1, -2)$ C. $(1, 2)$ D. $(-1, 6)$

9.3 DSE Corner

9.3.1 Long Questions

Problem 9.1 — HKDSE/2012/I/Q5.

There are 132 guards in an exhibition centre consisting of 6 zones. Each zone has the same number of guards. In each zone, there are 4 more female guards than male guards. Find the number of male guards in the exhibition centre.

9.3.2 Multiple Choice Questions

Problem 9.2 — 2012/II/Q5.

If $m + 2n + 6 = 2m - n = 7$, then $n =$

A. -4. B. -1. C. 3. D. 11.

Problem 9.3 — 2014/II/Q8.

The price of 2 bowls and 3 cups is \$506. If the price of 5 bowls and the price of 4 cups are the same, then the price of a bowl is

A. \$88. B. \$92. C. \$110. D. \$115.

Problem 9.4 — 2015/II/Q3.

If $p + 3q = 4$ and $5p + 9q = 2$, then $p =$

A. -5. B. -3. C. 3. D. 5.

Problem 9.5 — 2016/II/Q5.

If $4\alpha + \beta = 7\alpha + 3\beta = 5$, then $\beta =$

A. -3. B. -2. C. 2. D. 3.

Problem 9.6 — 2019/II/Q3.
If $6x-7y=40=2x+11y$, then $y=$

A. -4. B. 2. C. 4. D. 9.

Problem 9.7 — SP/II/Q4.
Let p and q be constants. If $x^2+p(x+5)+q\equiv(x-2)(x+5)$, then $q=$

A. -25. B. -10. C. 3. D. 5.

Problem 9.8 — PP/II/Q4.
Let m and n be constants. If $m(x-3)^2+n(x+1)^2\equiv x^2-38x+41$, then $m=$

A. -4. B. -1. C. 3. D. 5.

Problem 9.9 — 2012/II/Q3.
Let p and q be constants. If $x^2+p\equiv(x+2)(x+q)+10$, then $p=$

A. -4. B. -2. C. 6. D. 10.

Problem 9.10 — 2013/II/Q8.
If a, b and c are non-zero constants such that $x(x+3a)+a\equiv x^2+2(bx+c)$, then $a:b:c=$

A. $2:3:1$. B. $2:3:4$. C. $3:2:6$. D. $6:4:3$.

Problem 9.11 — 2014/II/Q3.
If p and q are constants such that $px(x-1)+x^2\equiv qx(x-2)+4x$, then $p=$

A. 1. B. 2. C. 3. D. 4.

Problem 9.12 — 2015/II/Q5.
If m and n are constants such that $x^2+mx+n\equiv(x+4)(x-m)+6$, then $n=$

A. -8. B. -2. C. 2. D. 6.

Problem 9.13 — 2017/II/Q8.
If m and n are constants such that $4x^2+m(x+1)+28\equiv mx(x+3)+n(x-4)$, then $n=$

A. -8. B. -7. C. 4. D. 16.

Problem 9.14 — 2019/II/Q4.
If α and β are constants such that $(x-8)(x+\alpha)-6\equiv(x-9)^2+\beta$, then $\beta=$

A. -26. B. -10. C. -7. D. -6.

Problem 9.15 — 2020/II/Q7.
If h and k are constants such that $(x+h)(x+6)\equiv(x+4)^2+k$, then $k=$

A. -28. B. -16. C. -4. D. 2.

Problem 9.16 — 2021/II/Q6.
If a, b and c are non-zero constants such that $a(x+3)+b(3x+1)\equiv c(x+2)$, then $a:b=$

A. $1:3$. B. $1:5$. C. $3:1$. D. $5:1$.

Problem 9.17 — 2022/II/Q3.
If m and n are constants such that $(x+3)^2+mx \equiv (x-n)(x+1)+2n$, then $m=$

A. -14. B. -8. C. 4. D. 9.

Problem 9.18 — 2023/II/Q5.
If c and d are constants such that $(x+2)(x+3)+12 \equiv x(x+d)+6c(x+1)$, then $d=$

A. -13. B. -3. C. 3. D. 17.

9.4 Calculator Program

?→A: ?→B: ?→C: ?→D: ?→X: ?→Y: AX – DB→M: M⁻¹(CX - YB→X ◢ M⁻¹(AY – DC→Y

Total Number of bytes: 53 bytes or 49 bytes (if you do not need to save the answers). To add the program, choose `MODE 6 1 (PROG NO)`, then start.

9.4.1 How to use the program

■ Example 9.2

$$\begin{cases} x+y &= 7 \\ x-y &= 1 \end{cases}$$

Choose `PROG (PROG NO)` to enter the program. Enter all coefficients. `1 EXE 1 EXE 7 EXE` (For the first equation), `1 EXE -1 EXE 1 EXE` (For the second equation) Then the program will output `(4) EXE (3) EXE`. The answer is then $x=4$ and $y=3$. The full progress shown as below (Program number = 1):

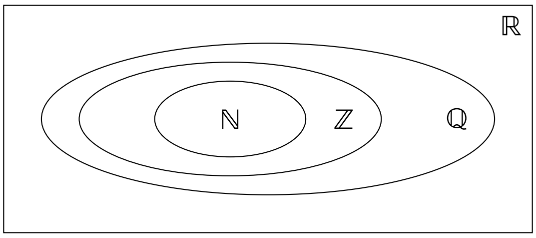
```
PROG 1 1 EXE 1 EXE 7 EXE 1 EXE -1 EXE 1 EXE (4) EXE (3) EXE
```

■

10. Pythagoras' Theorem and Irrational Numbers

10.1 Rational and Irrational Numbers

Before, we had learnt natural numbers, integers, fractions and decimals. In Mathematics, they will be grouped as different sets, a group of numbers. We group them into natural numbers ($\mathbb{N}$), integers ($\mathbb{Z}$) and rational numbers ($\mathbb{Q}$). All of them are under the group of real numbers ($\mathbb{R}$). Other than rational numbers, there is something called irrational numbers. Then how do we know when we should call it as rational and when call it as irrational? We need the definitions.



Definition 10.1 — Rational Number.
Any rational numbers can be expressed as a form of $\frac{m}{n}$, where m, n are integers and $n \neq 0$.

R All natural numbers, integers, fractions, terminating decimals and recurring decimals are rational numbers.

Definition 10.2 — Irrational Number.
An irrational number is a real number which cannot be expressed as a form of $\frac{m}{n}$, where m, n are integers and $n \neq 0$.

R In Form 4, you will know what is a "not real" number (complex number).

In irrational numbers, we have two types, algebraic and transcendental. One of the transcendental numbers you have learnt is π. Another example is e, which you will learn if you take M2. Algebraic irrational numbers are normally involves *surd*, the reverse operation of *power*.

In Mathematics, a root means the solution to an equation. For example, in $x^2 = a$, x is a root to the equation. Another way to say is x is a square root of a. Symbolically, $x = \sqrt{a}$. The symbol

"$\sqrt{\ }$" is a *radical* sign. Each positive number a has two square roots, $\sqrt{a}$ and $-\sqrt{a}$ or in short $\pm\sqrt{a}$. What if a is a negative number? You will learn it in Form 4. *Surds* are irrational square roots.

R If $n^2 < a < (n+1)^2$, where n is a positive integer, then $n < \sqrt{a} < n+1$.

There are some operations about surds. Such as $\sqrt{ab} = \sqrt{a}\sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. Also, to remove the radical sign of denominator (or numerator), we normally we multiply and divide the same number in denominator (or numerator). This process is called rationalisation of denominator (or numerator). There will be a more advanced technique in rationalisation in M2.

■ Example 10.1

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$ ■

However, additions and subtractions are not that freely with surds. $\sqrt{a} \pm \sqrt{b} \neq \sqrt{a \pm b}$. It is better to consider them as some algebra, where like terms can do additions and subtractions, while unlike terms cannot.

■ Example 10.2

$$\sqrt{3}+2\sqrt{12} = \sqrt{3}+2\sqrt{4\times 3} = \sqrt{3}+2\sqrt{4}\sqrt{3} = \sqrt{3}+4\sqrt{3} = 5\sqrt{3}.$$ ■

Exercise 10.1 Find the value of x in $x^2 = 81$.

Exercise 10.2 Which of the following number does not lie between 3 and 4?

A. $\sqrt{15}$ B. $\sqrt{13}$ C. $\sqrt{11}$ D. $\sqrt{8}$

Exercise 10.3 Find the value of $\sqrt{64 \times 25}$.

Exercise 10.4 Find the value of $\sqrt{\frac{27}{147}}$.

Exercise 10.5 Find the value of $\sqrt{16ab} \cdot \sqrt{ab^3}$.

Exercise 10.6 Rationalise the denominator of $\frac{2}{\sqrt{13}}$.

10.2 Pythagoras' Theorem and its converse

Pythagoras[1]discovered the Pythagoras' Theorem, which is one of the fundamental theorems in Geometry.[2]

[1] In Ancient Greeks, Pythagoras, born in 570 BC, died in 495 BC, was the first one to suggest this world is virtual.

[2] One fun fact about Pythagoras is that after he suggested the Pythagoras Theorem, one of his students Hippasus considered the existence of $\sqrt{2}$, an irrational number, from a right angled triangle with the lengths of two sides are both 1. Pythagorean thinks all numbers are rational numbers. The findings shocked Pythagoras. The next day after the question arised someone found the dead body of Hippasus on a beach.

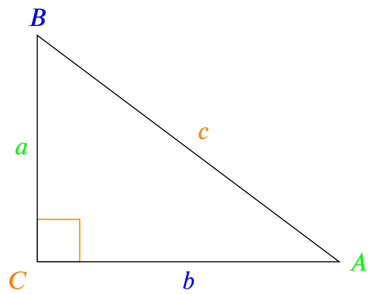
Theorem 10.1 — Pythagoras' Theorem (Abbreviation: *Pyth. theorem*).
If $\triangle ABC$ is a right-angled triangle, where $\angle C = 90°$, then the sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c). Symbolically, $a^2 + b^2 = c^2$.

There are many (more than 400) ways to prove the theorem. Proofs can be easily found online.

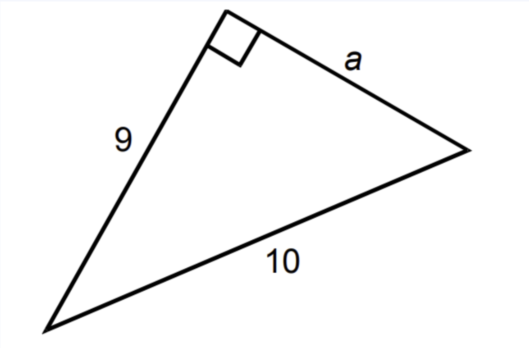
One may think if the converse is also true. In fact it is also true. Then in this situation, we would call it as "*if and only if*" or in short *iff* or symbolically $\Longleftrightarrow$.

Theorem 10.2 — Converse of Pythagoras' Theorem (Abbreviation: *converse of Pyth. theorem*).
If the sum of the areas of the two squares on the legs (a and b) equals the area of the square on the hypotenuse (c), $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right-angled triangle, where $\angle C = 90°$.

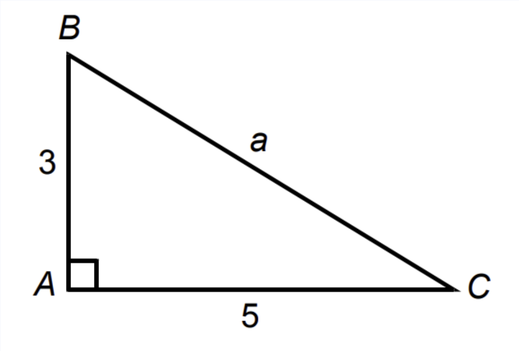
R We would usually denote one side with the opposite angle. For example, a to $\angle A$, b to $\angle B$ and c to $\angle C$.



Exercise 10.7 In the figure, find the value of a.



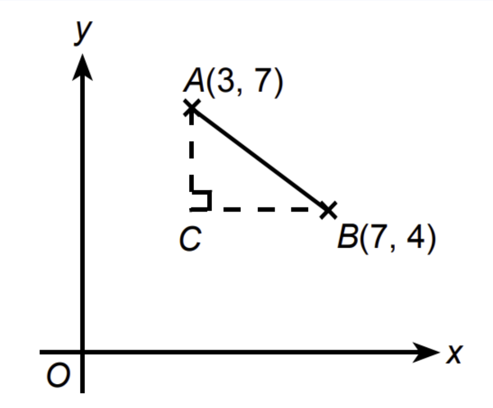
Exercise 10.8 In the figure, find the value of a.



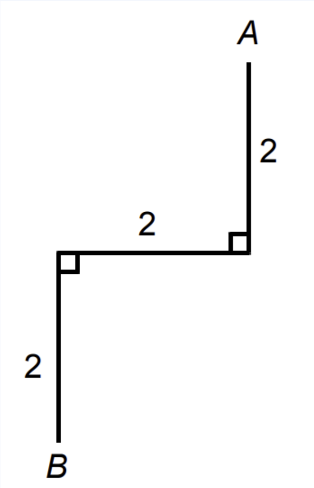
Exercise 10.9 If x, y and z are the lengths of the sides of a right-angled triangle where $z < y < x$, which of the following is true?

A. $x^2 = z^2 - y^2$ B. $y^2 = z^2 - x^2$ C. $z^2 = x^2 - y^2$ D. $z^2 = y^2 - x^2$

Exercise 10.10 In the figure, find the length of AB.

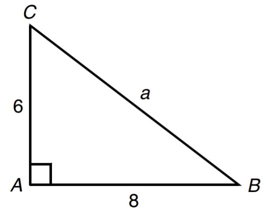


Exercise 10.11 In the figure,find the length of AB.

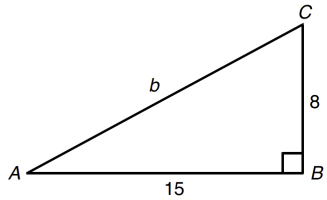


Exercise 10.12 Find the values of the unknown marked sides.

(a)

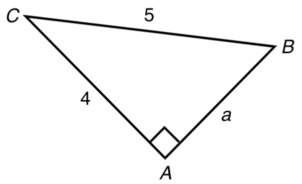


(b)

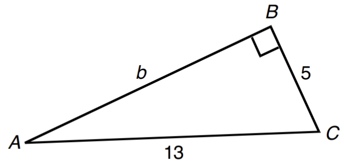


Exercise 10.13 Find the values of the unknown marked sides.

(a)

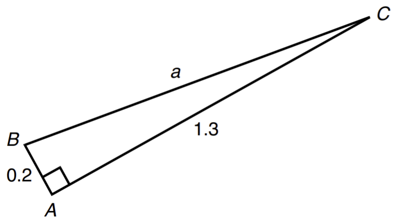


(b)

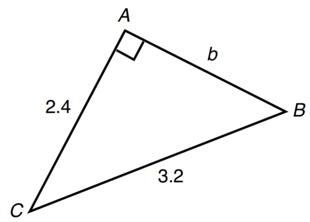


Exercise 10.14 Find the values of the unknown marked sides.

(a)

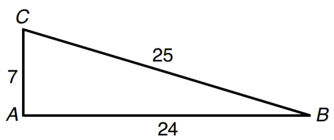


(b)

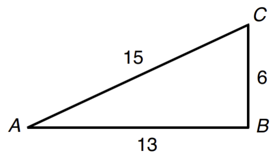


Exercise 10.15 Determine whether the following triangles are right-angled triangles. State the right angle for each right-angled triangle.

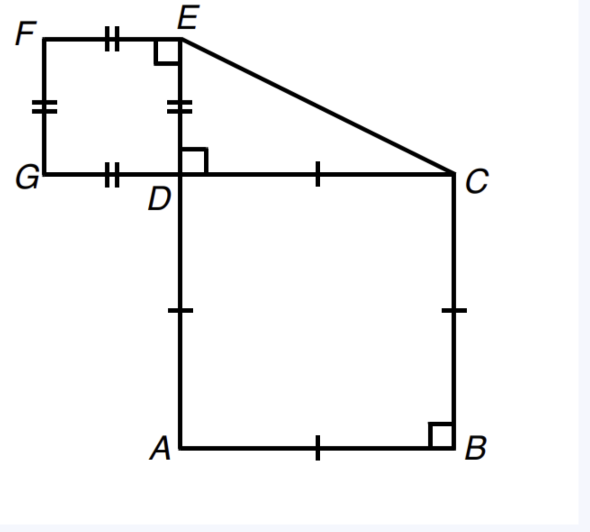
(a)



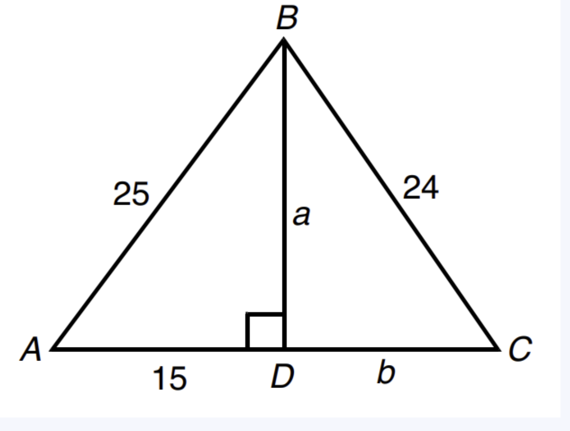
(b)



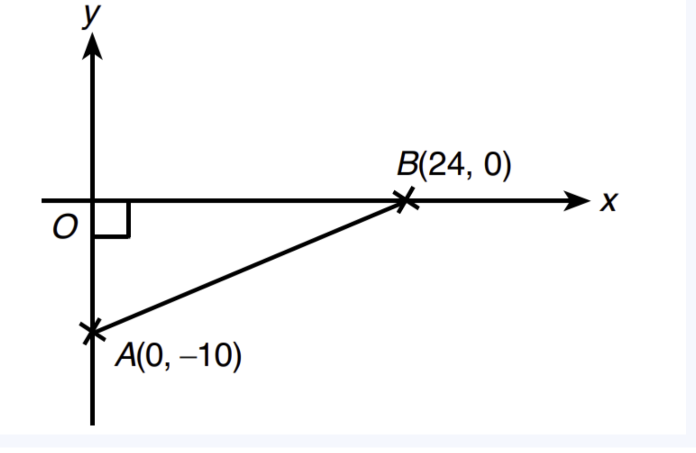
Exercise 10.16 In the following figure, the areas of the squares $ABCD$ and $DEFG$ are 144 cm^2 and 25 cm^2 respectively. Find the length of EC.



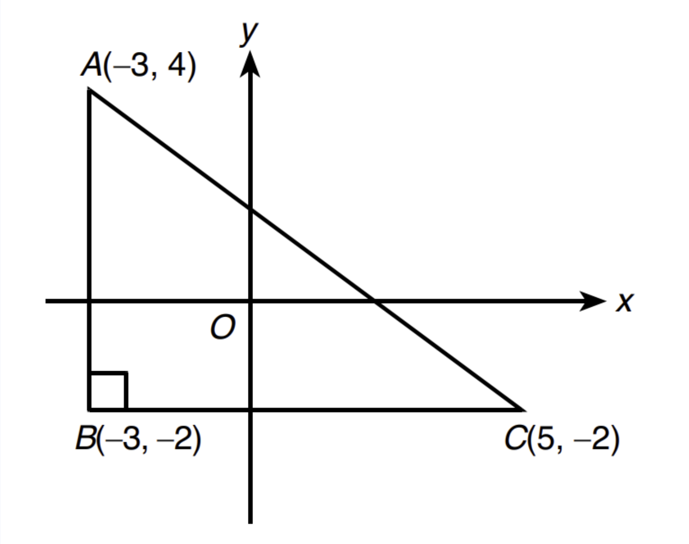
Exercise 10.17 In the figure, ADC is a straight line. Find the unknowns.



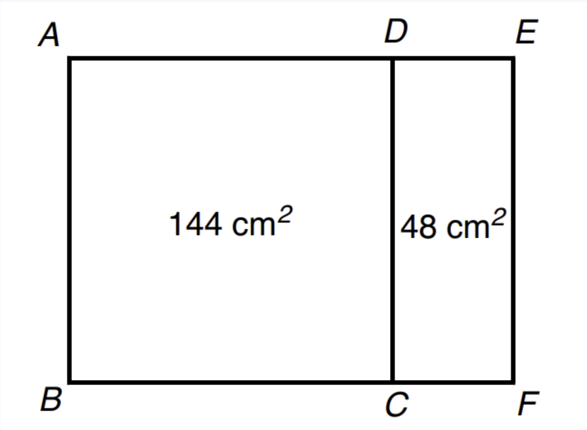
Exercise 10.18 In the figure, find the length of AB.



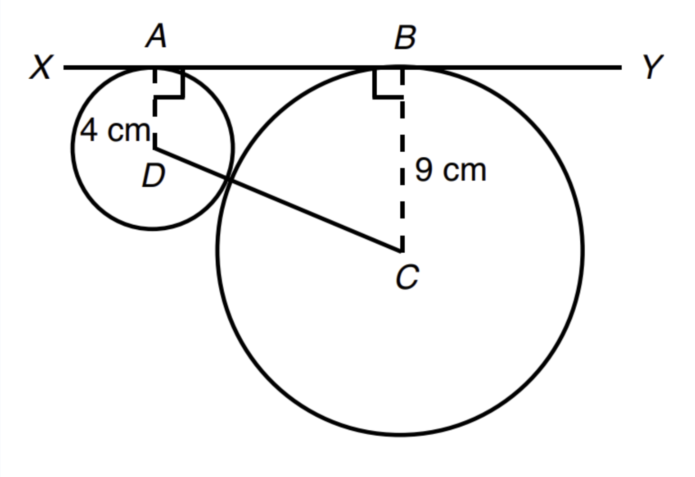
Exercise 10.19 In the figure, $A(-3,4)$, $B(-3,-2)$ and $C(5,-2)$ are the vertices of right-angled triangle ABC. Find the perimeter of $\triangle ABC$.



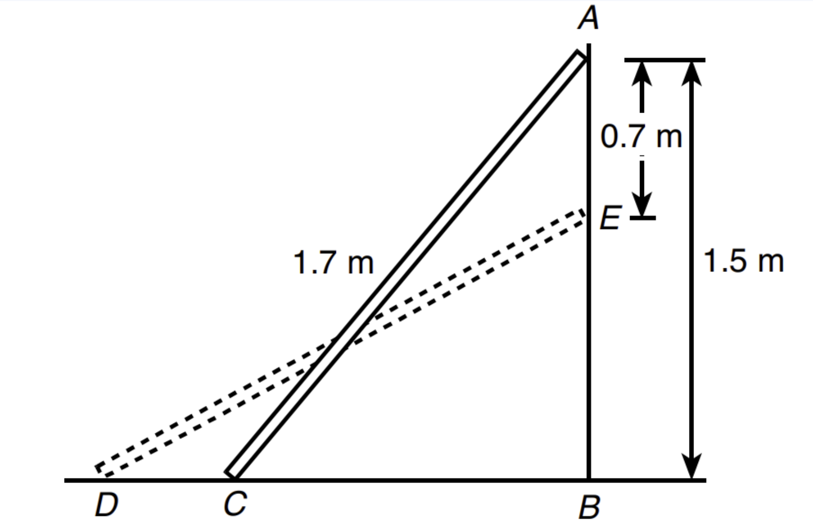
Exercise 10.20 The figure shows rectangle $ABFE$. It is formed by square $ABCD$ with area 144 cm^2 and rectangle $DCFE$ with area 48 cm^2. Find the length and width of rectangle $ABFE$.



Exercise 10.21 In the figure, C and D are the centres of two circles. The line XY touches the circles at A and B respectively. If $AD \perp XY$, $BC \perp XY$, $AD = 4$ cm and $BC = 9$ cm, find the length of AB.



Exercise 10.22 A ladder of 1.7 m leans against a vertical wall AB. The top of the ladder is 1.5 m above the ground. If the top of the ladder slides down 0.7 m such that the foot of the ladder slides to position D as shown, find the distance DC.



11. Areas and Volumes (II)

11.1 Circumferences and Areas of Circles

11.1.1 Circumferences of Circles

We have learnt the ratio of the *circumference* (C) to the *diameter* (d) of a circle is π, which is $C = \pi d = 2\pi r$, where r is the radius of the circle.

Exercise 11.1 Find the circumference of a cricle with diameter 20 cm.

Exercise 11.2 Find the diameter of a cricle with circumference 20 cm.

Exercise 11.3 Find the circumference of a cricle with radius 20 cm.

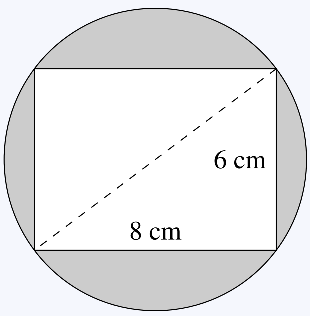
11.1.2 Areas of Circles

The area (A) of a circle is $A = \pi r^2$.

Exercise 11.4 Find the area of a circle with radius 10 cm.

Exercise 11.5 Find the circumference of a circle with area 10 cm^2.

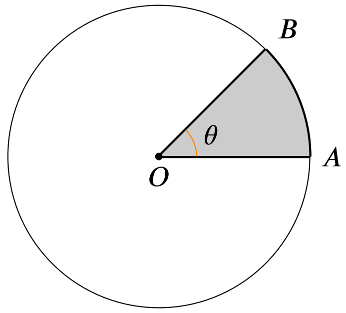
Exercise 11.6 Find the area of the shaded region.



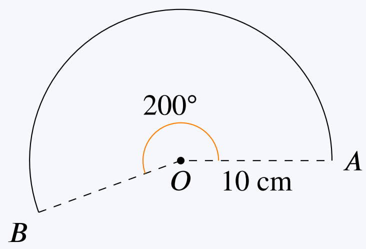
11.2 Lengths of Arcs and Areas of Sectors

11.2.1 Lengths of Arcs

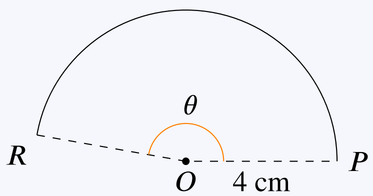
The curve AB is called an *arc*. It can be denoted by "arc AB" or "$\overset{\frown}{AB}$". $\angle AOB$ is the angle at the centre. The arc length $= \frac{\theta}{360°} \times 2\pi r$.



Exercise 11.7 Find the length of $\overset{\frown}{AB}$.

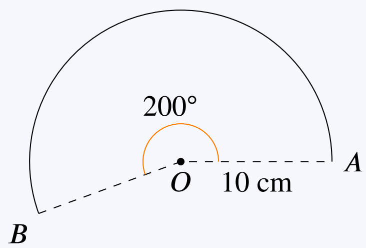


Exercise 11.8 If $\overset{\frown}{PR} = 12$ cm, find θ.

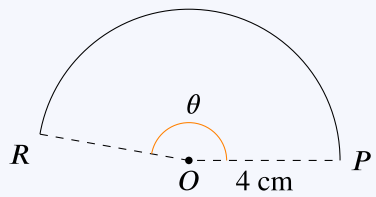


In the same graph, the filled part is called a *sector*. $\angle AOB$ is also called the *angle of the sector*. The area of the sector $= \frac{\theta}{360°} \times \pi r^2$.

Exercise 11.9 Find the area of the sector OAB.



Exercise 11.10 If the area of sector OPR is 24 cm, find θ.



11.3 Cylinders

11.3.1 Volumes of Cylinders

Similar to prism, volume of a cylinder is the base area × height, which is $V = \pi r^2 h$.

Exercise 11.11 Find the volume of a cylinder with radius 0.5 m and height 3 m.

Exercise 11.12 The base radius and the height of a cylinder are r m and 3 m respectively. If the volume of the cylinder is 15 m^3, find the value of r.

11.3.2 Total Surface Areas of Cylinders

Similar to prism, the "lateral" surface area of a cylinder, or the curved surface area, is the perimeter of the base × the height. So we have

$$\text{Curved surface area of a cylinder} = 2\pi rh,$$
$$\text{Total surface area of a prism} = 2\pi rh + 2\pi r^2.$$

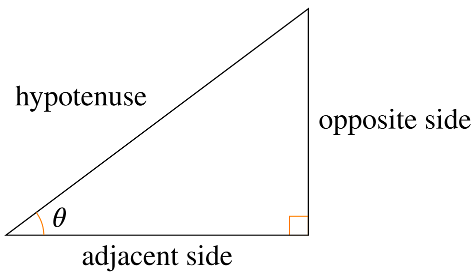
Exercise 11.13 Find the total surface area of a cylinder with radius 0.5 m and height 3 m.

Exercise 11.14 The height and the base area of a cylindrical compact disc box are 3 cm and 49π cm^2 respectively. Find the curved surface area of the compact disc box.

12. Trigonometric Ratios

12.1 Introduction to Trigonometric Ratios

We first have to discuss the terms in a right-angled triangle.



Definition 12.1 — Sine Ratio.
The sine ratio is the ratio of the opposite side of θ to the hypothenuse.

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Definition 12.2 — Cosine Ratio.
The sine ratio is the ratio of the adjacent side of θ to the hypothenuse.

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

Definition 12.3 — Tangent Ratio.
The sine ratio is the ratio of the adjacent side of θ to the opposite side of θ.

$$\tan\theta = \frac{\text{adjacent side}}{\text{opposite side}}$$

If given any two side, we can find the angle back. The function used could be arcsine, arccosine or arctangent. Symbolically, $\arcsin$, $\arccos$, $\arctan$; or $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$. Here the $^{-1}$ does not mean the reciprocal of the functions but the inverse of the function. To prevent ambiguity, either we don't write them or use arc symbol.

There are two properties of sine ratio in a right-angled triangle.

1. $0 < \sin\theta < 1$
2. The value of $\sin\theta$ increases as the acute angle θ increases. (Note that it is not linearly related.)

There are two properties of cosine ratio in a right-angled triangle.

1. $0 < \cos\theta < 1$
2. The value of $\cos\theta$ decreases as the acute angle θ increases. (Note that it is not linearly related.)

There are two properties of tangent ratio in a right-angled triangle.

1. $\tan\theta > 0$
2. The value of $\tan\theta$ increases as the acute angle θ increases. (Note that it is not linearly related.)

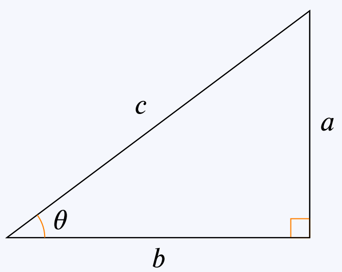
Exercise 12.1 Find the values of the following expressions.

1. $5\cos 45° + 2\sin 60°$
2. $7\sin 30° - 3\tan 15°$
3. $4\cos 75° + 6\tan 50°$
4. $2\sin 90° - 5\cos 30°$
5. $8\tan 20° + 3\sin 40°$
6. $10\sin 10° - 4\cos 80°$
7. $6\cos 60° + 2\tan 70°$
8. $3\sin 45° + 5\cos 30°$
9. $9\tan 25° - 2\sin 50°$
10. $1 + 4\cos 15° + 3\tan 10°$

Exercise 12.2 Find the values of the following expressions.

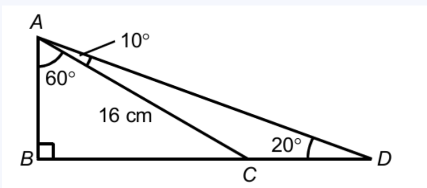
1. $12\sin\theta = 5$
2. $5\cos\theta = 4$
3. $7\tan\theta = 3$
4. $\sin\theta = 2\tan 20°$
5. $\cos\theta = 4\tan 15°$
6. $\tan\theta = 5\sin 30°$

Exercise 12.3 In the figure, $\cos\theta =$

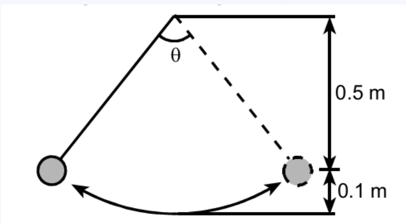


Exercise 12.4 If $\cos\theta = \cos 72° - \frac{1}{4}\cos 32°$, find θ.

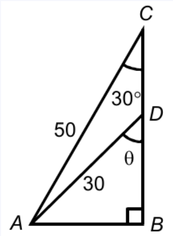
Exercise 12.5 In the figure, BCD is a straight line. Find AD.



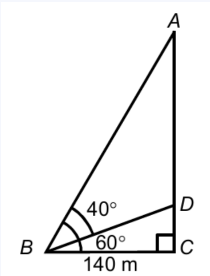
Exercise 12.6 A pendulum swings back and forth as shown in the figure. Find θ.



Exercise 12.7 In the figure, CDB is a straight line. Find θ.

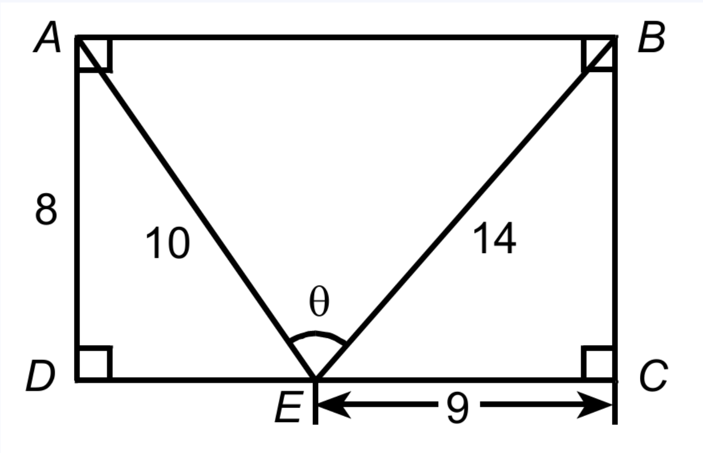


Exercise 12.8 The following figure shows a right-angled triangle ABC where $\angle ABC = 60°$, $\angle ABD = 40°$. Find AD.

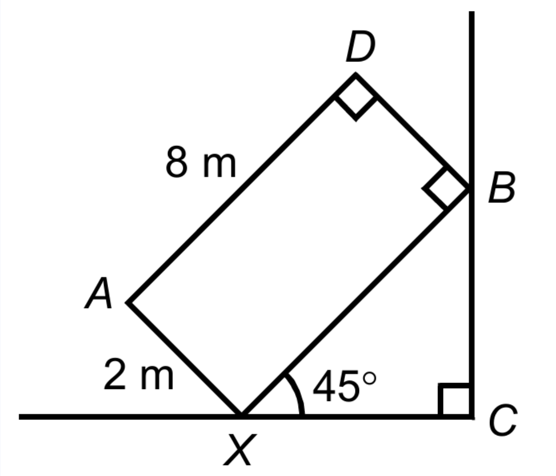


12.2 Simple Applications of Trigonometric Ratios

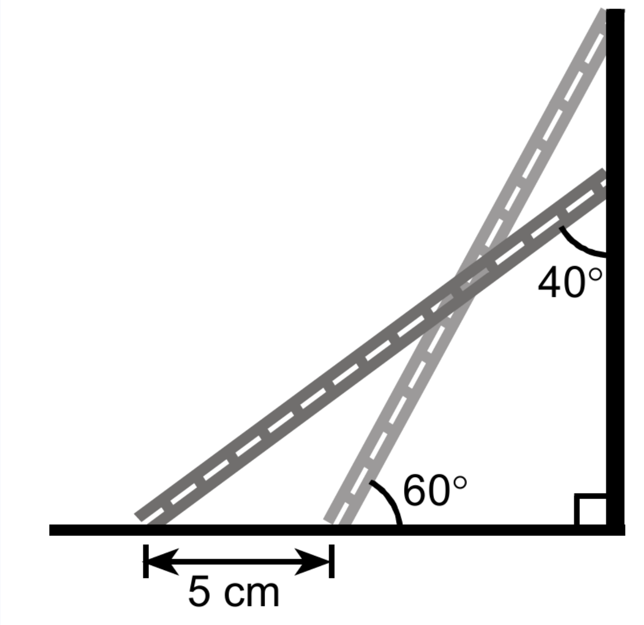
Exercise 12.9 The figure shows a rectangle ABCD. Find θ.



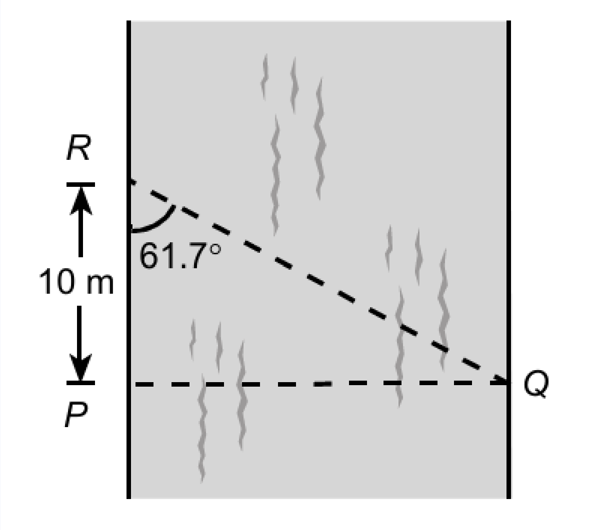
Exercise 12.10 In the figure, $ADBX$ is a rectangle. Find the distance between point D and the ground XC.



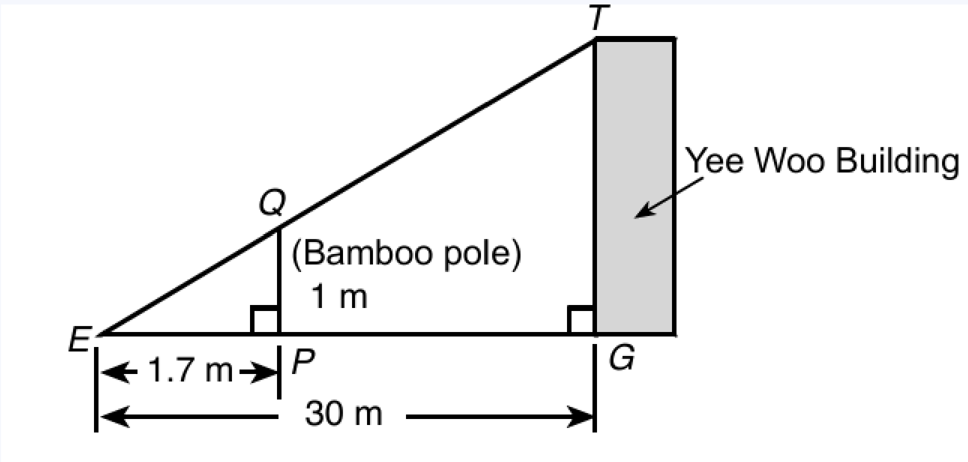
Exercise 12.11 In the figure, a ladder leans against a vertical wall. When the foot of the ladder slides 5 cm to the left, how far does the top of the ladder move vertically?



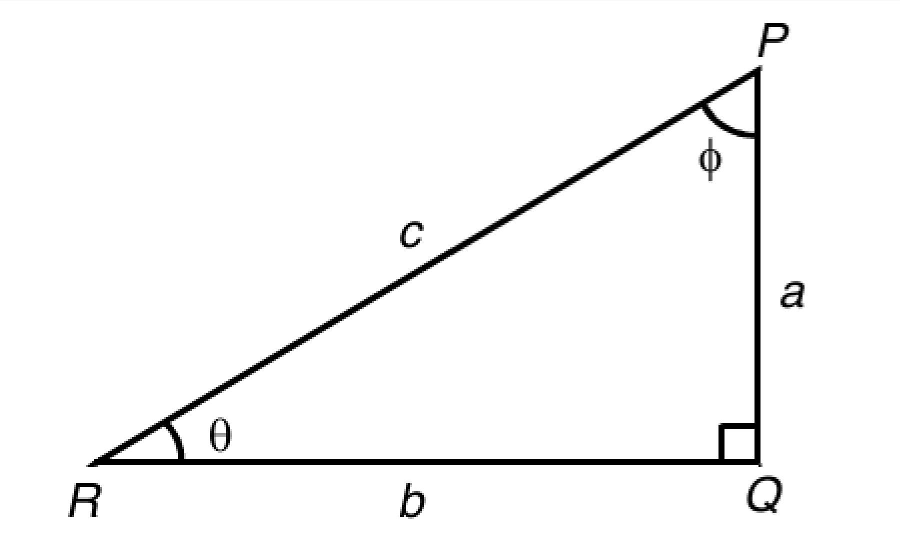
Exercise 12.12 Students A, B and C try to measure the width of a river by using a measuring tape of 10 m long and a protractor. Student A stands at point P, student B stands at point Q, and student C uncoils the measuring tape and reaches point R so that $PR = 10$ m and $\angle QPR = 90°$. If $\angle PRQ$ is measured to be 61.7°, find the width of the river.



Exercise 12.13 Firefly and Kafka try to measure the height of Yee Woo Building by using a measuring tape and a bamboo pole. Firefly lies on the floor at point E which is 30 m from the building. She looks at the top T of the building. Kafka moves the bamboo pole from point E towards the building until the tip of the bamboo pole Q, points E and T form a straight line. If $EP = 1.7$ m and the length of the bamboo pole is 1 m, find the height of Yee Woo Building.



Exercise 12.14 The figure shows $\angle PQR$ where $PQ = a$, $QR = b$, $PR = c$, $PQ \perp QR$, $\angle PRQ = \theta$, $\angle RPQ = \phi$.



(a) Express $\sin\theta$, $\cos\theta$ and $\tan\theta$ in terms of a, b and c.
(b) Express $\sin\phi$, $\cos\phi$ and $\tan\phi$ in terms of a, b and c.
(c) Find $\theta + \phi$.
(d) Using the results of (a), (b) and (c),
  (i) find the relationship between $\cos(90° - \theta)$ and $\sin\theta$.
  (ii) express $\tan(90° - \theta)$ in terms of $\tan\theta$.
(e) Using the result of (a), find the value of $\sin^2\theta + \cos^2\theta$.